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The identification of dominant macroeconomic drivers: coping with confounding shocks

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Abstract

We address the identification of low-frequency macroeconomic shocks, such as technology, in Structural Vector Autoregressions. Whilst identification issues with long-run restrictions are well documented, we demonstrate that the recent attempt to overcome said issues using the Max-Share approach of Francis et al. (2014) and Barsky and Sims (2011) has its own shortcomings, primarily that they are vulnerable to bias from confounding non-technology shocks, although less so than long-run specifications. We offer a new spectral methodology to improve empirical identification. This new preferred methodology offers equivalent or improved identification in a wide range of data generating processes and when applied to US data. Our findings on the bias generated by confounding shocks also importantly extends to the identification of dominant business-cycle shocks, which will be a combination of shocks rather than a single structural driver. This can result in a mis-characterization of the business cycle anatomy.

Keywords: Identification, Long-Horizon and Business-Cycle Shocks, Confounding Shocks.

JEL classification: C11, C30, E32

Non-technical summary

Structural Vector Autoregressions (SVARs) have increasingly been used to understand the dominant drivers of both business-cycle and long-term fluctuations in the macroeconomy. Understanding these drivers can provide useful insights into important questions, such as whether long-term productivity improvements come at the short-term cost of lower employment, or whether the business cycle is predominantly driven by demand-side or supply-side factors.

This paper notes several theoretical shortcomings of existing SVAR identifications of dominant drivers of business cycle and long-run economic fluctuations. This paper documents the biases that can be introduced into the long-run and variance-maximizing methodologies by confounding shocks, i.e. shocks other than the target of interest. Three alternative SVAR identifications are proposed to deal with confounding shocks with different frequency domain properties, two of which are new (Limited Spectral and NAMS). For identifying long-run shocks such as new technological innovations, all proposed approaches and the Max-Share methodology are found to be more robust to multiple sources of bias compared to the widely used "long-run" identification of technology shocks of Galí (1999).

Two types of SVAR identifications in the frequency domain show a further reduction in estimation bias when identifying long-term shocks such as technology in the presence of business-cycle frequency confounding shocks. The newly developed Limited Spectral identification also reduces lag-truncation bias compared to existing spectral methodologies to a degree. These sources of bias are shown to exist in data for the United States. The Spectral identifications are the only methods able to both robustly estimate the SVAR using multiple data transformations suggesting that they are the most appropriate identification method of long-term economic drivers such as technology shocks.

This paper also illustrates a key weakness of variance-maximizing SVAR identifications of dominant business-cycle shocks, such as the spectral approach at business-cycle frequencies. We show that the identified shock will be a variance-weighted combination of many different business-cycle drivers. Using a simple model, we show that recent research that has identified a non-inflationary "main business cycle" shock is likely to instead be capturing a combination of inflationary New-Keynesian style demand-drivers and deflationary positive supply shocks. The paper demonstrates that caution should be applied in interpreting the results of variance-maximizing methods in the frequency domain.

1 Introduction

In this paper we revisit the use of Structural Vector Autoregressions (hereafter, SVAR) to identify dominant shocks, focusing on, *but not limited to*, its specific application to the identification of technology shocks.[1] We examine the recently proposed solutions to address known drawbacks of the traditional long-run restriction by Francis et al. (2014) and Barsky and Sims (2011), through medium-run variance-maximizing identifications, and unearth a key weakness of these methodologies that have so far been under-explored but which have important consequences; notably, that these methodologies are likely to capture a range of shocks in addition to the target of interest. We suggest further modifications to the existing methods and point out the features of the data generating process that will result in the choice of one modification over another. In addition, these methods are also being used to identify dominant drivers of business-cycle macroeconomic developments (Angeletos and Dellas, 2020; Levchenko and Pandalai-Nayar, 2018). We show that these applications can also suffer the same drawbacks due to confounding shocks, resulting in misleading conclusions being drawn.

The use of SVARs to 'look-through' cyclical changes in productivity and isolate structural developments, or changes in technology, can be traced back to Blanchard and Quah (1989). In their approach, long-run restrictions are imposed in a structural VAR to separate the effects of temporary 'demand' and permanent 'supply' shocks on GDP. This methodology was later adapted by Galí (1999) to specifically identify technology shocks in a two-variable VAR containing log-differences of productivity and hours worked.

Long-run restrictions have been criticized on two main grounds, one economic and the other econometric: first, it is restrictive to assume that technology is the only shock that can affect productivity in the long-run;[2] and second, econometrically, that imposing long-run restrictions on a finite sample and in the presence of non-technology shocks leads to biased and inefficient estimates. It is the second strand of the literature we investigate more deeply in this paper, notably the pitfalls of using alternative medium-run restrictions to identify long-run shocks. The

[1] We follow the literature and use "technology" as a catchall phrase for the dominant long-run shock, while "non-technology" represents the shocks with more fleeting effects.

[2] Several strands of research have suggested other non-technology shocks that may also result in permanent effects on labor productivity. Mertens and Ravn (2013) find that changes in taxation can have long-running effects of productivity, which once controlled for, lead to different dynamics of macroeconomic variables in response to identified technology shocks. Fisher (2006) separates productivity shocks into those that specifically apply to investment goods (IST shocks), and neutral technology shocks that affect aggregate production. In addition, Francis and Ramey (2005) and Uhlig (2004) have debated the extent to which the presence of these additional shocks alters the literature's findings on the impact of technology shocks on hours worked.

medium-run identification strategy is more robust to estimation on finite samples, identifying technology shocks as those which contribute the most to the forecast error variance decomposition (FEVD) of labor productivity at the 10-year horizon (Francis et al., 2014). We hereafter refer to this approach as the 'Max-Share' identification. This strategy has also been used to identify technology news shocks, which reflect changes in the perception of future long-run technology developments (Barsky and Sims, 2011).

An important gap in the literature is how to identify technology shocks in the presence of confounding shocks, a key feature of the data. One significant exception to this is Chari et al. (2008), who find that long-run restrictions are only unbiased when technology shocks dominate as a driver of output and non-technology shocks play a small role in a DSGE setting. In circumstances where non-technology shocks play a material role in driving macroeconomic variables, long-run identifications will not only be biased, but will often show small and significant confidence intervals around biased results — they will be 'confidently wrong' about the impact of technology shocks. There is evidence that non-technology shocks are likely to play a larger role in output fluctuations than commonly assumed, with estimates for the contribution of technology shocks to output ranging from just 2 to 63% of output variability (Galí and Rabanal (2005), Christiano et al. (2003), Chari et al. (2008)). The issues raised by Chari et al. (2008) for long-run identifications have yet to be raised for newer alternatives.

We show that the Max-Share approach overcomes many of the weaknesses of long-run restrictions but can be biased by confounding non-technology shocks just as long-run identifications can. We consider alternative approaches to sharpen the identification of technology shocks in the presence of confounding shocks. The first, Non-Accumulated Max-Share (NAMS), shows a lower degree of bias in the presence of low-frequency confounding non-technology shocks but is susceptible to lag-truncation and short-sample bias. We next propose a spectral identification approach whose rotation is in the frequency domain. Finally, we modify the spectral approach by substituting the long-run variance-covariance matrix with one that can be reasonably obtained in small samples. This preferred new methodologies is more robust to confounding shocks than the Max-Share approach and is more robust to lag-truncation bias than existing methodologies used to identify dominant shocks in the frequency domain (Angeletos and Dellas, 2020; DiCecio and Owyang, 2010).

We highlight that the influence of confounding shocks and lag-truncation bias can have very different effects even between two standard and relatively small-scale DSGE models. As an alternative, we employ a simple two-variable model that provides greater transparency around the data-generating process than a DSGE model and removes the influence of lag-truncation bias to demonstrate how the impact of confounding shocks can be dependent on their nature.

Our findings have important implications for the identification of dominant business-cycle as well as long-run macroeconomic drivers. Variance-maximising identifications have recently been used to deconstruct the anatomy of business cycles. Specifically, Angeletos and Dellas (2020) applied a similar identification concept at business cycle frequencies in an effort to determine the possibility of a single shock driving the real side of the economy with little inflationary impact, in contrast to the standard New Keynesian paradigm. We argue that confounding issues are more acute at business-cycle frequencies and demonstrate the challenges they pose in determining the true business cycle anatomy.

The remainder of the paper is as follows: In section 2, we first revisit existing technology identification methodologies, drawing attention to newly identified shortcomings of the Max-Share and long-run approaches before outlining several proposed improvements. In section 3, we then show the performance of each identification methodology in Monte Carlo simulations on DSGE-generated data. We perform various experiments to identify the influence of varying the importance of confounding shocks on each methodology, as well as the influence of short-sample and lag-length limitations. In section 4, we simplify the data generating process by using a simple 2-variable model to isolate the effects of different types of confounding shocks: whether they drive low or business-cycle fluctuations in the data. This simplified model does not have an ∞-representation, removing the influence of lag-truncation bias. In section 5, we then take each specification to data for the United States, assessing which sources of bias are likely impacting the identifications. Finally, we switch focus to demonstrate how confounding shocks paint a different anatomical picture of business-cycle drivers than that proposed by recent macroeconomic literature.

2 Empirical Approaches

In this section, we briefly revisit the standard long-run and Max-Share identification of a technology shock. The potential for the Max-Share approach to be contaminated by other non-technology shocks is then explained, before we detail three approaches that can reduce contamination, depending on its source.

2.1 Identifications and their Drawbacks

2.1.1 Long-Run Restrictions

We start with the simple and original approach that first introduced SVARs as a tool to disentangle technology shocks from general macroeconomic fluctuations (Galí, 1999). Isolating the long-run components of labor productivity ($prod_t$) and total hours worked ($hours_t$), labeled LP_{LR} and $Hours_{LR}$, respectively, this methodology imposes the restriction that only the technology shock can impact labor productivity in the long-run.

$$\begin{Bmatrix} LP_{LR} \\ Hours_{LR} \end{Bmatrix} = \begin{Bmatrix} * & 0 \\ * & * \end{Bmatrix} \begin{Bmatrix} \epsilon_{tech.} \\ \epsilon_{non-tech.} \end{Bmatrix} \tag{1}$$

Assuming the structural AR matrix polynomial,

$$A\left(L\right) = I_2 - A_1 L - A_2 L^2 \ldots - A_p L^p \tag{2}$$

The long-run counterpart is therefore,

$$A\left(1\right) = \; I_2 - A_1 - A_2 \ldots - A_p \tag{3}$$

In a stationary VAR containing the log-difference series of productivity and hours, the long-run effect of the technology shock on growth will dissipate. The long run impact of each shock on the level of the target variable can be written as:

$$\begin{bmatrix} LP_{LR} \\ Hours_{LR} \end{bmatrix} = A(1)^{-1} \begin{bmatrix} \epsilon_{tech.} \\ \epsilon_{non-tech.} \end{bmatrix} = B(1)^{-1} A_0^{-1} \begin{bmatrix} \epsilon_{tech.} \\ \epsilon_{non-tec.} \end{bmatrix} = \begin{bmatrix} \Theta_{11} & 0 \\ \Theta_{21} & \Theta_{22} \end{bmatrix} \begin{bmatrix} \epsilon_{tech.} \\ \epsilon_{non-tech.} \end{bmatrix} \tag{4}$$

where $B(L)$ is the reduced-form VAR polynomial. Restricting the loading of the non-technology shock onto productivity to be zero can be accomplished by ensuring the long-run impact matrix is lower triangular. This is accomplished by solving for A_0^{-1} as follows:

$$A_0^{-1} = B(1) chol[B\left(1\right)^{-1} \Sigma_u B(1)^{-1'}] \tag{5}$$

Where Σ_u is the reduced-form variance-covariance matrix.

2.1.2 Max-Share

Long-run restrictions have come under fire for short-sample estimation problems and overly restrictive assumptions. The Max-Share identification instead assumes that technology shocks are the *dominant* driver of productivity around the 10-year horizon. In this identification, the technology shock is that which drives the largest proportion of the forecast error variance of labor productivity at this horizon, as in (Francis et al., 2014). When employing Max-Share one has to be cognizant of the fact that at long horizons there are (cumulative) errors in the MA coefficients on which the identification rests. These errors can be sizeable as argued in Lemma 1 below.

Lemma 1 *VARs can produce increasingly biased and uncertain impulse responses as the horizon increases. A simple argument illustrates this issue: For*

$$y_t = \phi y_{t-1} + \epsilon_t \tag{6}$$

the j^{th} MA coefficient (impulse response) is $\Psi_j = \phi^j$. This is a convex function of ϕ so Jensen's Inequality implies that $E(\hat{\phi}^j) > [E(\hat{\phi})]^j$. Even in cases where the OLS estimate of ϕ is biased downward $E(\hat{\phi}) < \phi$ due to small-sample bias (Kilian, 1998), the implied estimate of Ψ_j could be biased upward $E(\hat{\phi}^j) > \phi^j$. As j gets large this bias can become enormous. Therefore, placing restrictions at long(er) or infinite horizons can lead to severely biased impulse responses. In addition, the uncertainty around estimates of Ψ_j will increase exponentially as the time horizon of interest, j, increases. The function for the variance of the estimates for Ψ_j is again an increasing function of j, $E(\frac{1}{N}\sum_i^N[\hat{\phi}_i^j - [E(\hat{\phi})]^j]^2)$, again due to Jensen's inequality.

Francis et al. (2014) surmise that 10 years is longer than the period over which the business cycle occurs (typically assumed to be 2-8 years), but short enough to reduce challenges related to estimation on a finite sample. This restriction is imposed in a VAR containing productivity, hours, consumption and investment as a share of GDP. The forecast error at horizon k can be written:

$$y_{t+k} - \hat{y}_{t+k} = \sum_{\tau=0}^{k-1} D^{\tau} u_{t+k-\tau} \tag{7}$$

Where D is a matrix of moving average coefficients. By defining an orthonormal matrix A_0

with columns α, and φ as a selection vector (size 1×n), we find the shock j which maximizes the contribution to the total forecast error variance of variable i at horizon k

$$\max f(\alpha) = \frac{\varphi_i' \left(\sum_{\tau=0}^{k-1} D^{\tau} \alpha \alpha\prime D^{\tau\prime}\right) \varphi_i}{\varphi_i' \left(\sum_{\tau=0}^{k-1} D^{\tau} \Sigma_u D^{\tau\prime}\right) \varphi_i} \; s.t. \;\; \alpha'\alpha = 1 \tag{8}$$

Where φ_i is an indicator vector that selects the i^{th} impulse response vector. The technology shock at this maximized value is then: $\epsilon_t^{tech} = \alpha' chol\,(\Sigma_u)^{-1} u_t$. Following Uhlig (2003), identifying the structural shock that maximizes the contribution to the forecast error variance of productivity is solved by identifying the eigenvector associated with the maximum eigenvalue of V_τ, where V_τ is the FEVD of the target variable based on reduced-form shocks, and the denominator of $f(\alpha)$.

$$V_\tau = \; \varphi_i' \left(\sum_{\tau=0}^{k-1} D^{\tau} \Sigma_u D^{\tau\prime} \right) \varphi_i \tag{9}$$

2.2 Alternative Approaches

A drawback of the Max-Share approach is that in addition to capturing the long-run shock of interest it also captures aspects of other shocks in the data. We propose three alternative empirical approaches that offer reduced interference from these confounding shocks in different circumstances.

We first formally demonstrate the confounding nature of other shocks when using the Max-Share identification. We present the key equations of the eigenvalue-eigenvector problem and refer the reader to Appendix Section 8.1 for further details.

Max-Share involves setting up the Lagrangian for V_τ:

$$L(\alpha) = \alpha'(V_\tau)\alpha - \lambda(\alpha'\alpha - 1)$$

whose first order conditions reduce to solving for the eigenvector associated with the largest eigenvalue of V_τ

$$V_\tau \alpha = \lambda \alpha$$

For a simple two variable VAR with the true structural coefficients of the form:

$$A_0\epsilon = \begin{bmatrix} A_{11}^l & A_{12}^h \\ A_{21}^l & A_{22}^h \end{bmatrix} \begin{bmatrix} \epsilon^l \\ \epsilon^h \end{bmatrix}$$

where we use l and h to characterize the first shock as a low frequency (technology shock) and the second as a high frequency (business cycle) shock. Given reduced form MA coefficient counterparts, D, the solution for α at horizon k will be of the general form:[3]

$$\tilde{\alpha}(k) = \begin{bmatrix} \frac{A_{11}^l + \sum_{\tau=1}^{k}(D_\tau^{11}A_{11}^l + D_\tau^{12}A_{21}^l)}{A_{12}^h + \sum_{\tau=1}^{k}(D_\tau^{11}A_{12}^h + D_\tau^{12}A_{22}^h)} \\ 1 \end{bmatrix}$$

In an attempt to isolate the low frequency technology shock, we can clearly see the potential contamination coming from the high frequency shock as α depends on the initial impact of the respective shocks of the target variable (1) ($\frac{A_{11}^l}{A_{12}^h}$), and their relative persistence: $\frac{\sum_{\tau=1}^{k}(D_\tau^{11}A_{11}^l + D_\tau^{12}A_{21}^l)}{\sum_{\tau=1}^{k}(D_\tau^{11}A_{12}^h + D_\tau^{12}A_{22}^h)}$. Essentially, the derived shock will not be of a 'pure' form, but rather a combination of shocks with the ratio dependent on their importance in driving the forecast error variance at the chosen horizon; if the high frequency shock has a sizeable initial impact this will contaminate the identification more the shorter the identification horizon, while too long a horizon will cause more uncertainty in the parameters. As the true form of A_0 is unobserved, the extent of this contamination in empirical applications is also unknown.

In general, there is no closed form solution to the eigenvalue-eigenvector problem and the above expression is for an extremely special case where such a solution exists.[4] *However, we consider this illustrative as the more complicated solution will exhibit the features of this restrictive expression.*

2.2.1 Non-Accumulated Max-Share (NAMS)

The standard Max-Share approach takes cumulative forecast errors up to time k: see equation 7. As indicated above, and expanded in the Appendix, we show that there may be instances where non-trivial proportions of the forecast error variance is driven by shocks of lower persistence than the shock of interest.

The first methodology we propose will sharpen the identification of shocks with long-run

[3] To satisfy the unit length restriction we will need to further normalize this by the length of the eigenvector.

[4] For example, assuming the variance-covariance matrix is commutative at each horizon.

impacts by reducing the weight given to less persistent processes than technology. For example, a high volatility AR(1) process with a small AR coefficient is also a predominantly low-frequency process, but has no meaningful lasting impact at the 10-year horizon. As an alternative to Max-Share, we propose finding the maximum of the square of the impulse response functions at a particular horizon k. Here we aim to find the shock which has the maximum effects at say period 40, ignoring all the previous horizon shocks. At this horizon, it may be expected that the effects of lower-persistence shocks will have dissipated. The NAMS approach is implemented in a similar way to the Max-Share approach, by solving:

$$\max f^k(\alpha) = \frac{\varphi_i' \left(D^{k-1} \alpha \alpha \prime D^{k-1\prime}\right) \varphi_i}{\varphi_i' \left(D^{k-1} \Sigma_u D^{k-1\prime}\right) \varphi_i} \ s.t. \ \ \alpha'\alpha = 1 \tag{10}$$

Kurmann and Sims (2017) also advocate reducing the impact of less persistent shocks in the Max-Share approach. In the Barsky and Sims (2011) identification, the forecast error variance under consideration is 'double-weighted'; the maximization is applied to the summed forecast error variance from periods 1 to k ($\max \sum_{i=0}^{k} f^i(\alpha)$, compared to the Francis et al. (2014) approach of $\max f^k(\alpha)$). Kurmann and Sims (2017) propose returning to the original identification of Francis et al. (2014), which maximizes a single forecast error variance at horizon k, finding that it helps sharpen the identification of technology news shocks. Our NAMS approach takes this to its logical conclusion, further reducing the distortion caused by transitory shocks.[5]

2.2.2 Spectral Identification

The NAMS approach deals with contamination from low-persistence, low-frequency processes. However, the Max-Share approach may also be contaminated by driving processes that occur at business-cycle and higher frequencies. Where the amplitude of these shocks are (coincidentally) high at the chosen target range (k), the NAMS approach may not fare well.

We investigate the use of identification in the frequency domain, which can maximize the share of variance explained only at frequencies that are of interest, excluding those that are not. Identifying technology shocks through restrictions that explain the majority of low (long-term) frequency volatility of productivity is a novel approach. However, this methodology has in the past been used to assess the types of shocks which drive the business cycle. For example,

[5] In addition, Uhlig (2004) suggests an identification in which non-technology shocks will have no effect after 10 years, such that technology shocks can be identified by a restriction such that only the technology shock has an impact on productivity at that horizon. Our approach differs in that we do not exclude other shocks from having an effect at this horizon, and instead look for the shock that *dominates* at 10-years.

Angeletos and Dellas (2020) find that a single shock drives the majority of the variance of a range of macroeconomic variables at business cycle frequencies. DiCecio and Owyang (2010) use the spectral approach to conduct an identification of technology shocks that is similar to what we propose in this article but do not evaluate its performance.[6] We effectively apply a band-pass filter using the reduced form coefficients of a VAR containing macroeconomic variables, identifying the spectral density of the variables within a particular frequency band. We then identify the technology shock by maximizing the variance of productivity explained at the desired frequency.

The spectral density of series Y at frequency ω can be written as a Fourier transform of its auto and cross covariances (γ) at lag τ:

$$S_{YY}\left(\omega\right) = \int_{\tau=-\infty}^{\infty} \gamma\left(\tau\right) e^{-i\tau\omega} \tag{11}$$

Therefore, once $\gamma(\tau)$ in known, the spectrum, $S_{YY}(\omega)$, can be straightforwardly calculated. The reverse also holds. That is, knowing the spectrum, $S_{YY}(\omega)$, leads to an easy computation of $\gamma(\tau)$ by using the inverse Fourier transform,

$$\gamma(\tau) = \int_{-\pi}^{\pi} e^{i\tau\omega} S_{YY}(\omega) d\omega \tag{12}$$

Setting $\tau = 0$, gives the variance of the time series Y.

$$\gamma(0) = \int_{-\pi}^{\pi} S_{YY}(\omega) d\omega \tag{13}$$

This means that the variance of Y is the integral of the spectrum over all frequencies, $-\pi < \omega < \pi$. This further indicates that the spectrum decomposes the variance of Y into components from non-overlapping frequencies. Therefore, similar to the Max-Share identification, spectral analysis allows us to gauge the importance of cycles at different frequencies to the variance of the series of interest. And importantly, we can remove unwanted frequencies from the maximization problem.

To employ this methodology we first need to uncover a VAR representation of the spectral density of Y. We start by writing the Wold representation of the VAR (assuming it is invertible):

[6] In addition, Christiano et al. (2006) find that by using a spectral estimator at frequency 0, Long-run restriction estimates prove less biased following a MCMC assessment of VAR performance.

$$Y_t = \left(I - \left(B_1 L + B_2 L^2 + \ldots B_p L^p\right)\right)^{-1} u_t = D u_t \tag{14}$$

By post-multiplying Y_t by $Y_{t-\tau}$ and summing across its lags (of τ periods), the series of auto and cross covariances can be discretely approximated by:

$$\sum_{\tau=-\infty}^{\infty} \gamma(\tau) = \sum_{\tau=-\infty}^{\infty} E Y_t Y_{t-\tau} = D \Sigma_u D' \tag{15}$$

Then, by writing $D\left(e^{-i\tau\omega}\right) = \left(I - \left(B_1 L e^{-i\omega} + B_2 L^2 e^{-i2\omega} + \ldots B_p L^p e^{-ip\omega}\right)\right)^{-1}$, the spectral density of Y can be written as a function of the reduced-form VAR coefficients.

$$S_{YY}(\omega) = D\left(e^{-i\tau\omega}\right) \Sigma_u D\left(e^{i\tau\omega}\right)' = \sum_{\tau=-\infty}^{\infty} \gamma(\tau) e^{-i\tau\omega} \tag{16}$$

To assess the spectral density within a frequency band, the spectral power can be integrated between the bounds of the frequencies of interest $[\underline{\omega}\,, \bar{\omega}]$

As in the case of the Max-Share approach, the shock which maximizes the contribution to the variance of productivity over this band is the eigenvector associated with the largest eigenvalue of the matrix $\int_{\underline{\omega}}^{\bar{\omega}} S_{YY}(\omega)$. To identify technology, the band of interest is restricted to frequencies below (longer-than) 10 years, to exclude business-cycle frequencies.

2.2.3 Limited-Horizon Spectral Identification

One criticism that can be leveled against the spectral identification approach is that the long-run VAR representation used to calculate the spectrum of the endogenous variables may be biased when estimated on a short sample of data and with limited lags.

Simulation exercises in section 3.2.2 demonstrate that biases in the impulse responses of long-run restricted models get larger and larger with the response period, even where large numbers of lags are included in the estimation. This is especially true for reasonably lagged VARs (10 lags or below; those typically estimated with macro-data). At shorter horizons the responses are closer to the truth. Our proposal, therefore, is to use projections extending to 10 years (as in the original Max-Share approach) rather than the long-run representation. This horizon is long enough to capture the theoretical underpinning of the structural identification but short enough to reduce the degree of ”lag-truncation” bias.

A “windowed” selection of autocorrelations is often used to estimate the spectrum of a series.

The same principle can be used in the spectral VAR identification process, effectively applying a truncated kernal estimation approach. For example, the infinite-MA representation, $D(e^{i\omega})$ can be replaced with a limited horizon series of impulse response coefficients.

$$D^k(e^{-i\tau\omega}) = \sum_{\tau=0}^{k-1} D_\tau e^{-i\tau\omega} \tag{17}$$

The same process used in the standard spectral approach is also used to identify the shock of interest. For example, identifying technology shocks as the dominant drivers of labor productivity over frequencies below 10 years. Additionally, as will be shown subsequent sections, in DSGE models with infinite-order representations, and seemingly the US data, short-samples of data and a limited number of lags can result in a larger bias to identifications that rely on the infinite-order MA representation. The Limited Spectral methodology, like the Max-Share, is less susceptible to this bias.

3 DSGE Model Evaluation

Traditionally, SVARs seeking to extract an unobservable shock, such as technology, have been evaluated on their performance using simulated data from a DSGE model (Barsky and Sims, 2011; Chari et al., 2009; Erceg et al., 2005; Francis et al., 2014). This is an intuitive exercise, given that the true underlying shock is known by construction. In this section, we test our estimators with two known data-generating processes, a New-Keynesian model and a Real Business Cycle model. We argue that many of these tests have been performed using calibrations of these DSGE models which fail to test for the influence of material confounding shocks needed to replicate key features of US macroeconomic data. We later show that existing methodologies perform poorly when these features are included in simulated data from a simple 2-variable model that provides greater transparency over the data-generating process compared to a DSGE model.

Each VAR specification is first tested on a standard medium-scale New Keynesian model, of the type proposed by Christiano et al. (2005), and used by Francis et al. (2014) and Barsky and Sims (2011) to evaluate their respective implementations of the Max-Share identifications.[7] This model contains features such as persistent consumption habits, investment adjustment costs, capital utilization, and partial price and wage indexation (see Appendix 8.2). In addition,

[7] In the Barsky and Sims (2011) 'news' implementation of the Max-Share identification, an additional 'news' shock is added to the standard medium-scale NK model, which affects technology with a lag of one period.

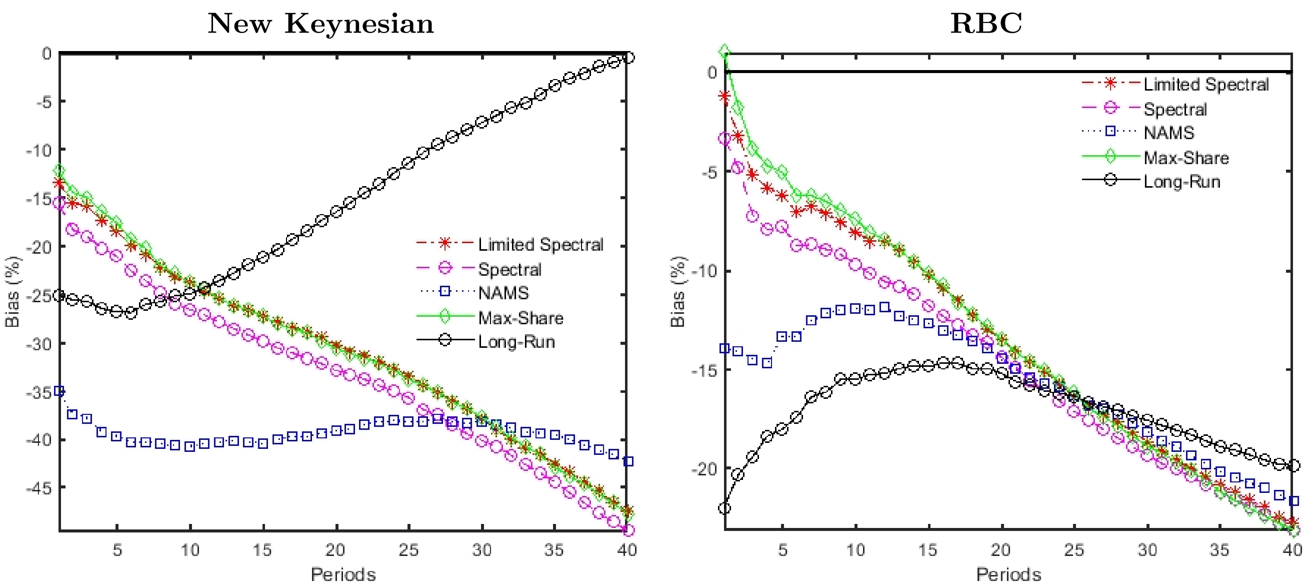
we also test the SVAR identifications on the RBC model used in Chari et al. (2008).

3.1 New Keynesian and RBC Monte-Carlo DSGE Results

A standard shock calibration is used to test each VAR specification on both the New Keynesian and RBC DSGE models. The first follows the calibration of Erceg et al. (2005), while the second follows the baseline calibration of Chari et al. (2008). The results below are based on 1000 simulations of each DSGE model, generating 250 observations after a burn-in of 100 periods. Each VAR is estimated via a Gibbs sampling procedure with flat priors, saving 1000 draws following a 500-period burn-in. The same procedure is used for all simulations throughout the document.

For the New Keynesian simulation, a four-variable VAR is estimated using: the natural logs of productivity, total hours worked, and the share of investment and consumption in GDP. For the RBC model, a two-variable VAR is estimated on the log-level of productivity and hours worked. For the long-run identification, log-differenced productivity is *always* used, as is standard. However, to demonstrate the effects of confounding shocks with different transformations of the data, we estimate the other SVARs (Max Share, NAMs, and Spectral methods) with productivity in both log-levels and log-differences.

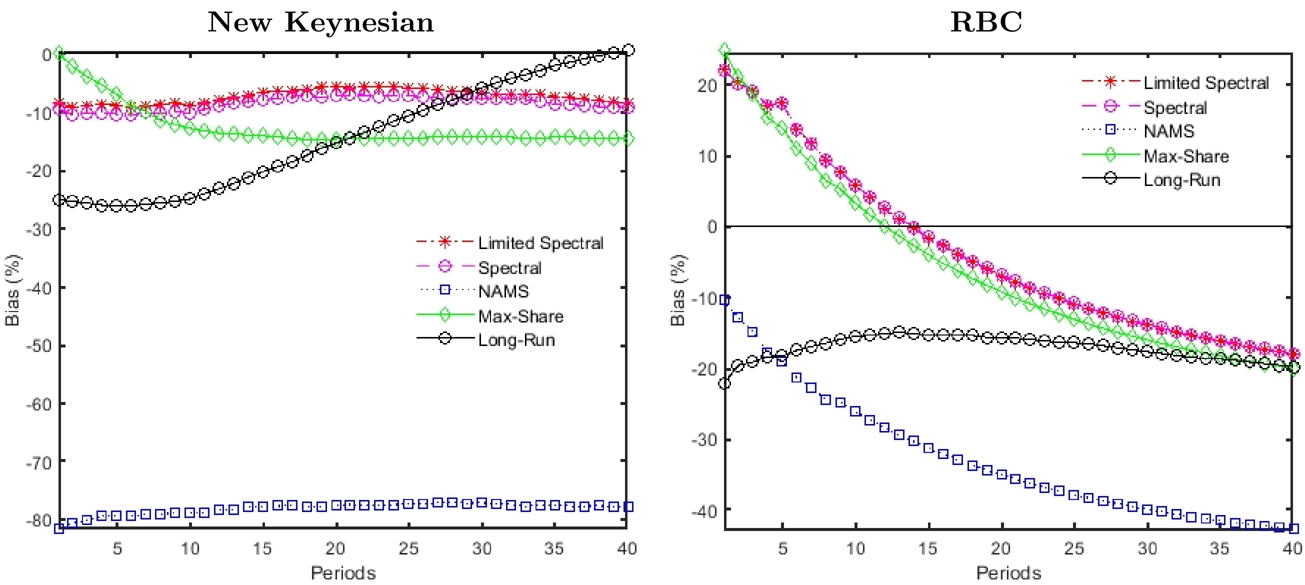
Figure 1: IRF bias of technology shock on productivity, estimation on log-level of productivity: NK and RBC models



Note: Long-run estimation performed using labor productivity in log-differences.

Labor productivity in levels: Figure 1 plots the biases of the impulse response of labor productivity to a technology shock when productivity is specified in levels (except the Long-Run SVAR). For both models, the Spectral, Limited Spectral, and Max-Share identifications have similar performances, showing the least bias for the initial response periods. However, their relative performance decays monotonically relative to NAMS. In both models NAMS has relatively high initial bias but its rate of decay is slower than that of the other estimators. The slower decay means that at longer horizons there is little distinction between the relative performance of all the estimators. At earlier horizons, the Limited Spectral and Max-Share identifications show slightly lower bias than the Spectral identification, which we will later argue is due to lower susceptibility to lag-truncation bias.

Figure 2: IRF bias of technology shock on productivity, estimation on differenced productivity: NK and RBC models



Labor productivity in log-differences: In the NK model, the Long-Run identification, estimated with productivity in log-differences, showed a lower degree of bias at medium to long horizons than the other estimation methods, estimated using the level of log-productivity (see left panel of Figure 1). However, when we estimate the models with productivity in differences for *all* specifications, there are clear patterns that emerge with respect to the degree of bias (see Figure 2). In both models, the Spectral, Limited Spectral and Max-Share again show similar level of bias across the response horizons. In the NK model, each of these identification have biases of the same sign and are best at recovering the true impulse responses of the DSGE models. However, in the RBC model, NAMS displays the least initial bias (in absolute value) but quickly decays for medium to long horizons. The four remaining methods (Max-Share, both Spectral, and Long-Run identifications) have similar absolute initial biases, negative for long-run and positive for the others. After horizon 30 all four display the same bias, in sign and magnitude.

Summary: The consistently high-performing SVARs in both simulations in the early periods of the IRF are the Max-Share, Spectral and Limited Spectral identifications. The shocks uncovered by the VARs have a high correlation (over 0.9) with the true underlying technology shock, corresponding to the low IRF bias for these specifications (Table 1). These three identifications also show lower degrees of bias in estimations of the impact of technology on hours-worked (Appendix 8.2). By period 30 of the IRF, the Max-Share, Spectral, Limited Spectral, and Long-Run identifications show similar biases in both models, with the exception of the Long-Run restriction in the New Keynesian case. When they are all estimated using labor productivity growth, the three aforementioned identifications outperform the Long-Run identification for at least the first 5-years of the response period. The NAMS specification has a higher IRF bias than the Max-Share, both Spectral specifications, and the Long-Run identifications in most cases. We surmise that this is mainly due to the fact that NAMS is estimated using parameters only at horizon 40 when the errors in estimation would have accumulated to sizeable degrees (see Lemma 1, and, as we will argue below, also due to the presence of lag-truncation bias).

Table 1: Correlation of VAR-identified technology shocks with true DSGE-generated shock

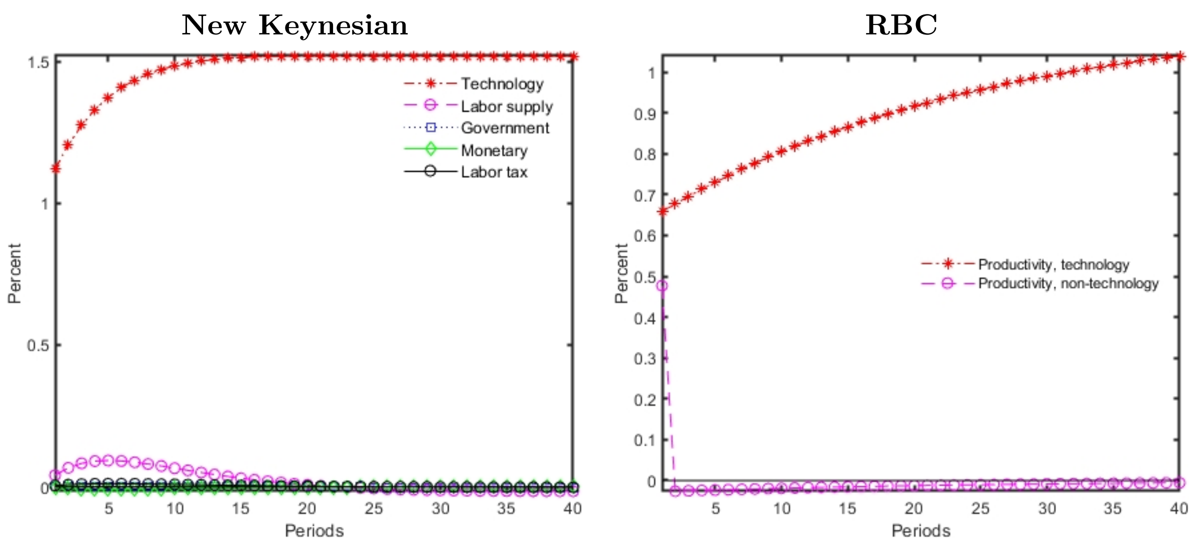
Identification	**Max-Share**	**Long-Run**	**Spectral**	**Limited Spectral**	**NAMS**
New Keynesian	0.90	0.85	0.90	0.90	0.77
	(0.70, 0.96)	(0.53, 0.96)	(0.64, 0.96)	(0.67, 0.96)	(0.43, 0.95)
RBC	0.97	0.86	0.96	0.97	0.95
	(0.86, 0.99)	(0.74, 0.94)	(0.84, 0.99)	(0.86, 0.99)	(0.80, 0.99)

Note: 5th and 95th percentile values shown in brackets.

The high performance of our estimators in levels in matching the true IRFs at early horizons is likely to be in part due to the shock processes driving the DSGE model, and the degree to which they drive the volatility of the simulated data (levels and differences). The highly persistent technology shock in this and other similar DSGE models tends to drive the vast majority of the variance of labor productivity. In this calibration, over 99% of the volatility of the level of labor productivity in both models is driven by the technology shock.[8] Therefore, several of the methodologies find a similar result given how insignificant the non-technology driving processes are in the variation of productivity.

The high performance of said estimators when using labor productivity growth is likely due to the technology shock lingering over time via the capital accumulation process. Notice that the relative performance of the three highest performing estimators (Max-Share, Spectral and Limited-Spectral) are even more pronounced in the NK DSGE model, where non-technology shocks display significantly smaller initial impacts on labor productivity (and therefore growth rates) than in the RBC specification (Figure 3). In contrast, the initial impact of the non-technology shock is almost as large as technology in the RBC model, resulting in larger biases.

Figure 3: Productivity IRFs for all shocks in New Keynesian and RBC models



When applied to US data we find that, unlike in the DSGE models, the impulse response of Max-Share differs significantly from those produced by the spectral methods when productivity is specified in differences. Our explanation of this finding rests on the DSGE models not containing the types of non-technology confounding factors as the data. This elucidates our earlier premise that Max-Share has even more difficulty (than spectral methods) disentangling technology shocks from persistent non-technology confounding shocks (see also section 4.1 for further illustration of this point).[9] *Therefore, when the nature of the confounding shock contributes non-trivially to*

[8] Based on re-simulating the model over 100,000 periods one shock at a time.

[9] See Sala (2015) who also estimate a New Keynesian DSGE model in the frequency domain. Also, Fernald (2007) found significant low frequency components in US productivity growth.

the dynamics of productivity the method used to identify technology becomes especially important.

Arguably, the standard DSGE specification does not adequately 'road-test' the performance of the VARs in a transparent way. In the following sections, we first examine VAR performance in the event that larger confounding shocks drive a more material component of the data process than assessed in traditional DSGE models. We address this issue by stripping away the complexities of the transmission mechanisms of our DSGE models and examine the performances of our SVAR estimators under a variety of non-technology confounding shocks in a simple two-variable setting. However, before doing so we address two important criticisms by Chari et al. (2008) (hereafter, CKM) levied against long-run identification using the RBC model, concerning the size of confounding shocks (but not their nature) and the impact of lag-truncation bias.

3.2 Addressing the CKM Critique of SVARs

In this section, we directly address two issues raised by Chari et al. (2008) about long-run identified SVARs: IRF bias with a varying importance of non-technology shocks, and IRF bias when the true lag length is infinity and is not imposed in estimation (lag-truncation bias). Chari et al. (2008) use the above-discussed RBC model to demonstrate the difficulties faced by long-run restricted SVARs in capturing technology shocks as other non-technology confounding shocks drive an increasing share of the variance in the model. We therefore test our proposed identification methodologies against this known DGP with which long-run restrictions are shown to have difficulty. We show that our new methodologies generally outperform the long-run identification SVAR, and perform comparably with the Max-Share identification in this RBC DSGE framework.

In this model, there is no advantage of using the new specifications above those offered by the Max-Share approach. The reason is that the non-technology shock has low-frequency properties (it is an AR(1) process), and therefore can confound the spectral as well as the Max-Share identifications. However, we demonstrate that compared to the long-run restriction approach, all other specifications show lower, or at least equal, degrees of bias. In subsequent sections, we show that there are circumstances where labor productivity is influenced by other types of confounding shocks where our new identifications outperform both the long-run and Max-Share identifications. Therefore, in contrast to the findings of CKM, when using the right estimator, SVARs can prove useful in identifying the impacts of technology shocks for a range of DGPs.

The RBC model is specified with a unit root for technology, but also a highly volatile and persistent non-technology shock τ, where it's persistence ρ_l is 0.95 in the standard calibration. Chari et al. (2008) note that two sources of bias exist for the long-run SVAR methodology: non-technology shocks will increase IRF bias as they drive a larger proportion of the variance of output; and lag-truncation bias, where limited VAR lags result in a bias due to the true specification of the VAR having an ∞-representation.

3.2.1 Varying the Relative Importance of Structural Shocks

Turning to the first claim, the relative variance of the non-technology shock to the technology shock is adjusted ($\frac{\sigma^2_{\epsilon_{lt}}}{\sigma^2_{z_t}}$) and the model is simulated 1000 for each ratio of variance, generating a data sample of 180 periods for each relative variance combination.[10] Each VAR is estimated using 4 lags. The long-run SVAR is estimated with log hours specified as $h_t - \alpha h_{t-1}$, where α determines the degree of quasi-differencing, as in CKM. This allows the VAR to be estimated with total hours in both levels and a highly quasi-differenced form.[11] All other SVAR specifications use hours in levels.

This exercise has a direct link to the eigenvalue-eigenvector rotation of variance-maximizing identifications. Recall the discussion in Section 2.2 where it was pointed out that success of Max-Share rotation, from reduced form to structural space, depends on the relative importance of the impact of the technology and non-technology shocks in driving the FEV at the targeted horizon.

Productivity in Levels:

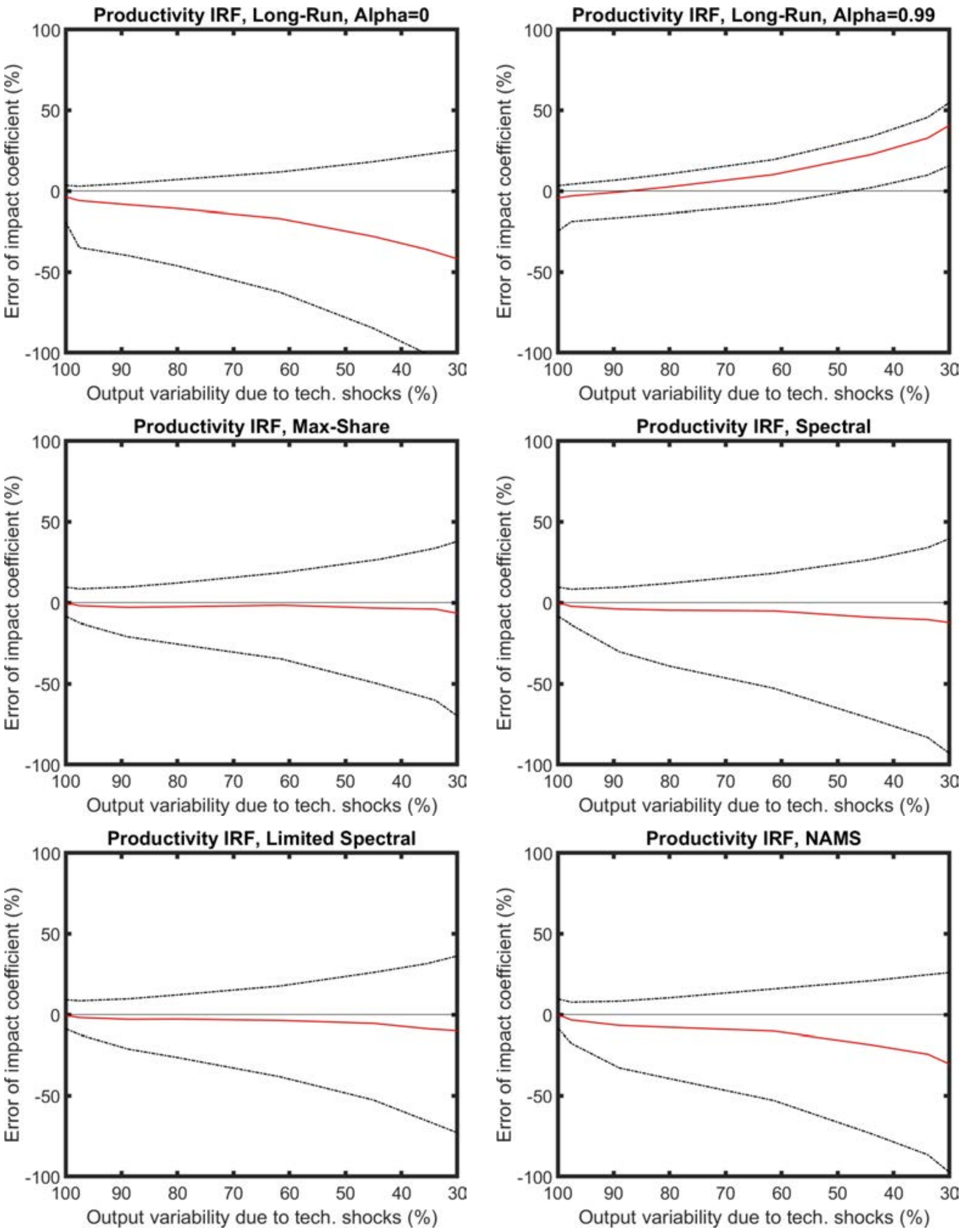
We draw the following conclusions from Figure 4:

- The long-run IRF for productivity is 'confidently wrong' (also demonstrated by Chari et al. (2008)) when the non-technology shock generates over 50% of the variance of output in the model in the quasi-differenced long-run specification. The specification with hours in levels has the largest bias and confidence intervals of the remaining specifications. However, the Max-Share and our new approaches *correctly* display uncertainty in the identification of technology shocks, via wider error bands, as the non-technology shock variance increases. See Christiano et al. (2007) for a discussion of large confidence intervals relative to the size of the bias in the context of Chari et al. (2008).

[10] The standard Gali calibration used to create Figure 1 in Chari et al. (2008) is used.

[11] Two specifications are used, with hours in levels ($\alpha = 0$) and hours with a high-degree of quasi-differencing ($\alpha = 0.99$).

Figure 4: CKM: Impact coefficient bias on labor productivity IRF as proportion of output driven by non-technology shock is varied

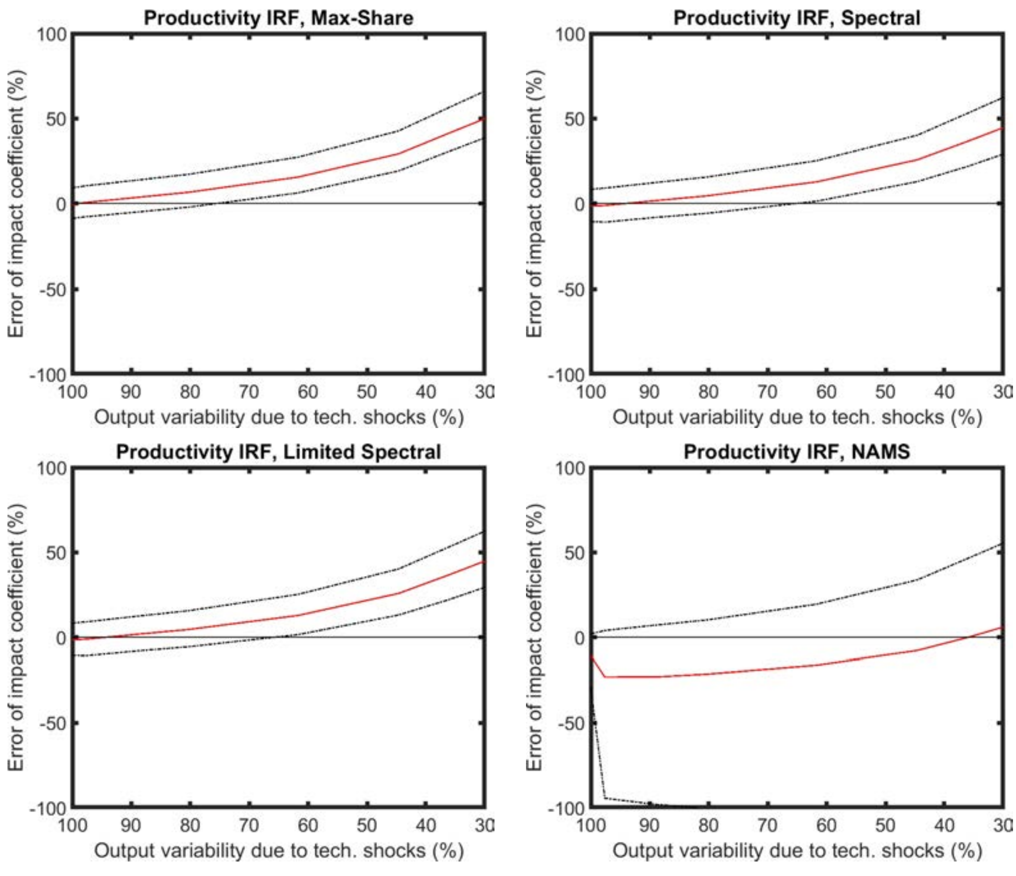


Note: The proportion of variance driven by the non-technology shock is calculated by simulating the model with one shock at a time, and then comparing the variance of the HP-filtered series for output from each simulation, as in CKM.

- The Max-Share, Limited-Spectral and NAMS approaches show smaller confidence bands than the Spectral methodology. These specifications do not rely upon the infinite MA representation, but instead use a finite representation. Their error bands are therefore smaller, reflecting a more efficient estimation.

- Our alternative methodologies show lower bias than the long-run identification as the non-technology shock increases in size. However, the Max-Share identification is marginally more efficient than the spectral approach, showing slightly less bias at all horizons. The NAMS identification becomes more biased as the non-technology shock grows larger in importance. Reducing the persistence of the non-technology shock relative to the CKM specification reduces this bias however (Appendix 8.4).

Figure 5: CKM estimated on differenced productivity: Impact coefficient bias on labor productivity IRF as proportion of output driven by non-technology shock is varied



Note: The proportion of variance driven by the non-technology shock is calculated by simulating the model with one shock at a time, and then comparing the variance of the HP-filtered series for output from each simulation, as in CKM. This chart shows the contribution of output variability to the level of labor productivity for consistency with figure 4

Productivity in Differences: In figure 5:

- The Max-Share and Spectral identifications show very similar degrees of bias to the highly quasi-differenced long-run specification. However, in section 4.1, we show that over longer horizons, beyond the initial impact periods, the Spectral approaches show considerably lower biases relative to the long-run and Max-Share identifications.

- The NAMS approach incorrectly captures the non-technology shock to a high-degree and a large number of draws show the effect on labor productivity falling close to zero. This can be intuitively understood: the effect of the technology shock on labor productivity *growth* is close to zero at the 10-year horizon, such that disentangling the two shocks is difficult for this methodology. In section 5, this is found to be a feature of the US data.

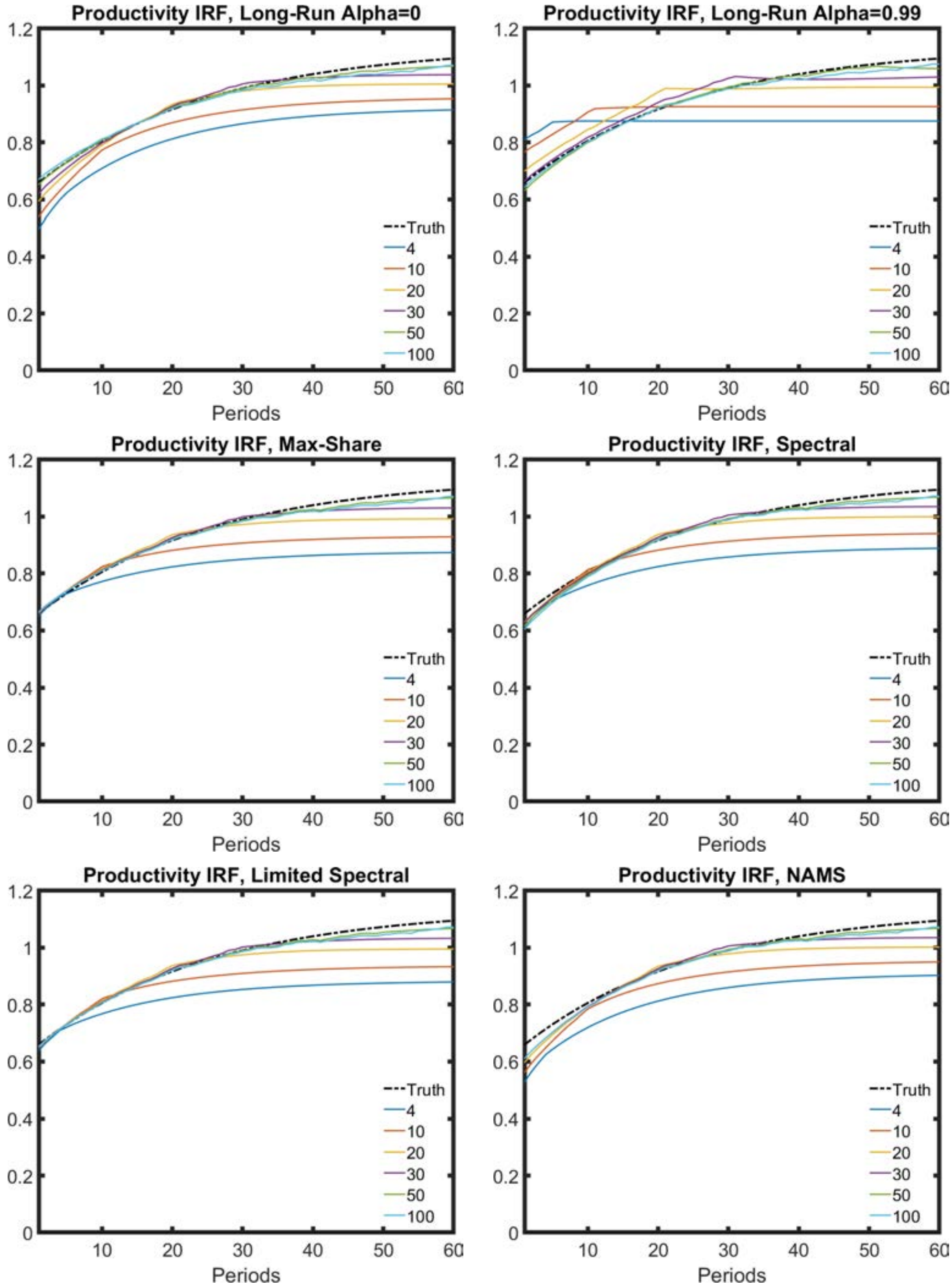
3.2.2 Lag-Truncation Bias

In the second exercise, we examine the robustness of each methodology to lag-truncation bias. This is the bias caused by estimating the VARs using a finite number of AR coefficients when the true DSGE-generated data has an infinite-lag order. There are two main findings from running each method on 100,000 simulated data points from the RBC model and varying the number of lags used in the estimation (Figure 6).

- At low lag levels, alternate methods show lower initial bias relative to the long-run specification. Further out, at long horizons, the new specifications show a similar bias to the long-run identifications.

- The NAMS and Spectral specifications continue to show more bias than the Limited Spectral and Max-Share specification on impact. The Spectral methodology is dependent on the long-run VAR representation, while the NAMS approach identifies the shock maximizing the variance at a long-term horizon (10-years), where the effects of lag-truncation bias are larger than in earlier periods of the IRF.

Figure 6: CKM: IRF bias as estimation lag-length is varied



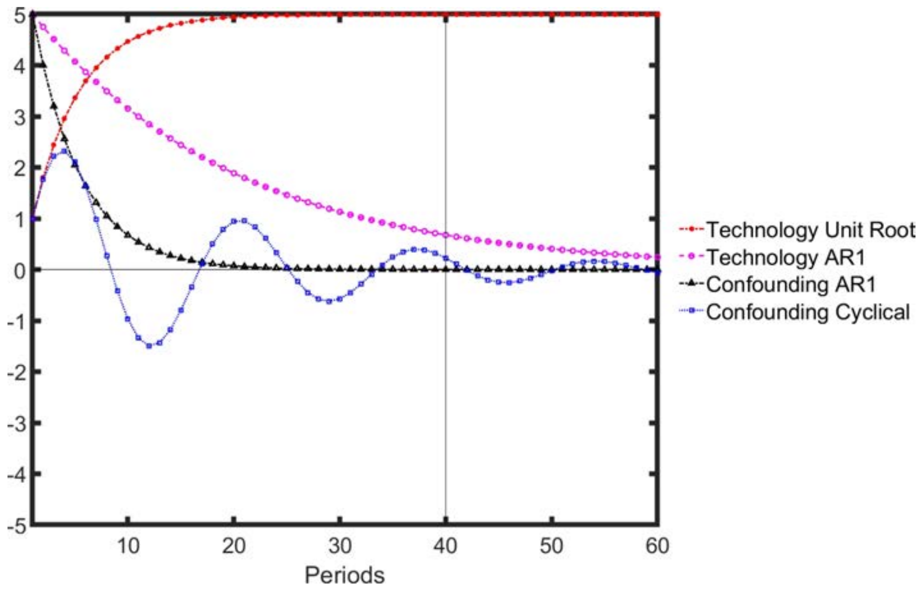
Note: Estimation on 100,000 periods of data simulated using the CKM RBC model, varying the number of lags used to estimate each SVAR. The relative variance of the technology and non-technology shocks are held constant, at $\frac{\sigma^2_{non-technology}}{\sigma^2_{technology}} = 0.64$, as in CKM.

4 An Illustrative Two Variable Data-Generating Process

Here we examine how the VARs perform when the data generating process is a more transparent set of technology and non-technology shocks. It is useful to strip away the complexity of the dynamics driven by the DSGE model so that we can clearly examine how different data processes can affect the results.

To that end, a simple two-variable model is used to generate the data. Figure 7 provides a stylized example of the different forms a traditional technology shock can take, from a persistent AR(1) to a unit-root shock. We test both forms in the scenarios below. In contrast, non-technology shocks to productivity, such as less persistent AR(1) and cyclical business-cycle-related shocks, may also be in the data, driving a material proportion of the variance. In a second simplifying assumption, the DGPs considered here do not have an infinite lag specification, therefore we abstract from the issues associated with lag-truncation bias.

Figure 7: Stylized IRFs for technology shock to productivity and confounding shocks



4.1 The Case of Unit Root Technology Shocks

We start by testing the identification of technology shocks in a simplified two-variable model where technology shocks take the form of a unit root process with a persistent growth component. This allows us to replicate the slow-building effect of technology shocks on labor productivity as in the DSGE models. This is also likely to be a key feature of the US data, as we will show. In contrast to the DSGE models previously explored, the confounding non-technology shock will be both high-volatility, and high (-er) persistence. This will allow us to replicate sharply differing results for the Max-Share and spectral methodologies that we find in US data.

A simple two-variable data process is generated for labor productivity (L) and hours (N). Both processes are driven by a technology shock (ϵ^{z^g}) and a non-technology shock (ϵ^b). The two-variable model takes the form:

$$L_t = z_t^l + b_t \tag{18}$$

$$N_t = 0.7N_{t-1} - 0.3N_{t-2} - 0.3z^l + 0.3b_t \tag{19}$$

$$z_t^l = z_{t-1}^l + z_t^g \tag{20}$$

$$z_t^g = \rho_{z^g} z_{t-1}^g + \epsilon_t^{z^g} \tag{21}$$

$$b_t = \rho_{b,1} b_{t-1} + \epsilon_t^b \tag{22}$$

The technology shock $\epsilon_t^{z^g}$ is a permanent shock to the level of productivity L, with persistent effects on its growth rate. ϵ_t^b provides a temporary impact on the level of productivity. We choose an illustrative calibration where the non-technology shock has a higher volatility but lower persistence than the technology shock ($\sigma^b = 2$, $\rho_b = 0.3$, $\sigma^{z^g} = 1$). The parameter ρ_{z^g}, which governs the persistence of the effect on productivity growth, is set to 0.8, a reasonable value.[12]

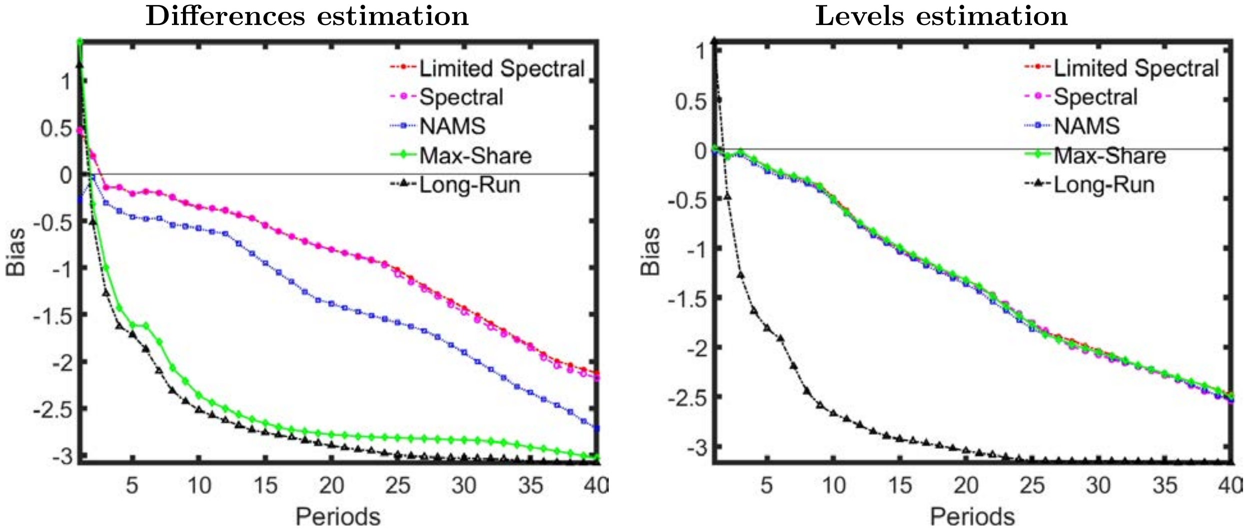
Given that we are interested in the particular case of a shock process driving the growth rate of productivity, we estimate each of the VARs in both levels and differences for productivity. In the first case, where the VARs are estimated on the differenced labor productivity series (L), the Spectral approaches show the lowest level of bias in their IRFs (Figure 8). In contrast, the Max-Share and long-run identifications show very high-levels of bias, taking on many of the properties of the non-technology shock.

To see why this is the case, observe that the differenced series L is the sum of the differenced series z^l and b ($\Delta L_t = \Delta z_t^l + \Delta b_t$). The first term is simply the low frequency AR(1) process

$$\Delta z^l = z_t^g = \rho_{z^g} z_{t-1}^g + \epsilon_t^{z^g}$$

[12] Lindé (2008) finds the persistence parameter to be low (0.14) but the variance of z^g to be high - he also finds the model fit was also very good when ρ_{z^g} was high but var(z^g) was low.

Figure 8: IRF bias where technology growth has a unit root



Note: The long-run specification requires the productivity data to be estimated in log differences. The results for the bias of the long-run specification in 'levels' plot reports the estimation in log differences for comparison purposes.

while the second reduces to

$$\Delta b_t = (\rho_b - 1)b_{t-1} + \epsilon_t^b$$

As $(\rho_b - 1)$ is negative, this second process is a mixture of high frequency and white noise processes. This may contribute to the volatility of ΔL but does not have persistent low-frequency effects. The Max-Share identification is, therefore, less capable of distinguishing between this and the true persistent technology shock. The Spectral approaches assign most weight to the low-frequency persistent shock, as does the NAMS approach, which 'looks through' the transitory white noise and high-frequency process resulting from differencing b. However, it is clear that the dominance of the technology shock in growth rates will fade as the persistence of the growth shock, ρ_{z^g}, falls. An exercise in which this parameter is reduced to 0.3 from 0.8 shows a severe deterioration in performance for the spectral approaches, although they continue to have the lowest bias on impact.

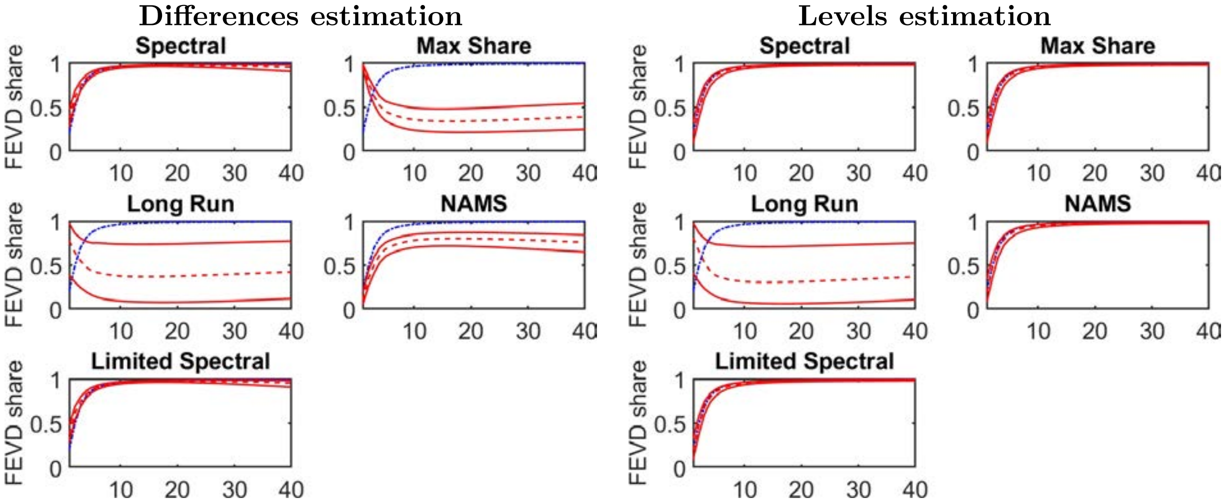
The ability of the Spectral and NAMS identifications to distinguish between these data generating processes also enables them to more accurately estimate the proportion of forecast error variance of productivity driven by the technology shock (Figure 9). When estimating the VAR in levels, we find that all approaches have a similar performance with the exception of the long-run restriction - which is always estimated in differences but shown for comparison

Table 2: Correlation of VAR-identified shocks with true technology shock when technology has a unit root

Identification	**Max-Share**	**Long-Run**	**Spectral**	**Limited Spectral**	**NAMS**
Differenced	0.31	0.23	0.96	0.96	0.96
	(0.10, 0.46)	(-0.04, 0.63)	(0.89, 0.98)	(0.89, 0.98)	(0.92, 0.98)
Levels	0.97	NA	0.97	0.97	0.97
	(0.93, 0.99)	NA	(0.93, 0.99)	(0.93, 0.99)	(0.83, 0.99)

Note: The long-run specification requires the productivity data to be estimated in differences in both cases. 5th and 95th percentiles shown in brackets.

Figure 9: Estimated FEVD where technology growth has a unit root



Note: 5th and 95th percentiles in brackets

purposes with levels results - (Figure 9 and Table 2). This accuracy is driven by all approaches accurately estimating the initial variance of the technology shock. However, the IRFs under all approaches are less accurate relative to those estimated by the Spectral VARs on the differenced data. Effectively, the additional dynamics in productivity growth driven by the technology shock are obscured when estimating the VAR with the data in levels. As such, the IRFs prove less persistent than the estimates of the Spectral and NAMS VARs on differenced productivity, and further away from the true persistence of the shock.

4.2 The Case of Stationary Technology Shocks

In the unit root case, the confounding non-technology shocks simply took the form of a less persistent, albeit also low-frequency shock. The form of the confounding shock was a secondary point, since the unit-root shock dominated the variance of observable variables, as in the New Keynesian and RBC models under consideration. There will however be situations in which the shock targeted by the econometrician is less clearly dominant in the DGP. This may occur for some economies subject to large non-technology shocks, or even when the econometrician is targeting shocks other than technology.

In this section, the model is further simplified so that the technology shock is no longer as clearly dominant in driving the variance of the model, and instead takes the form of an AR(1) process. The form of the confounding shock gains importance in these simulations. We now allow our confounding non-technology shock to take two forms: a low-frequency shock of the same form as the technology shock, but less persistent, and a second business-cycle frequency process.

Again, a simple two-variable data process is generated for labor productivity (L) and hours (N). Both processes are driven by a technology shock (ϵ^z) and a business-cycle shock (ϵ^b).

$$L_t = z_t + b_t \tag{23}$$

$$N_t = 0.7N_{t-1} - 0.3N_{t-2} - 0.3z_t + 0.3b_t \tag{24}$$

$$z_t = 0.9z_{t-1} + \epsilon_t^z \tag{25}$$

$$b_t = \rho_{b,1}b_{t-1} + \rho_{b,2}b_{t-2} + \epsilon_t^b \tag{26}$$

This simple process is calibrated to replicate some of the features of a more complex model, while being more transparent. In the case of a technology shock, labor productivity rises persistently, while hours-worked initially falls (as in the New Keynesian framework). The advantage of this simple setup is that we can change the driving processes of the non-technology shock through the ρ_b parameters and easily understand how this changes the properties of the data and hence the estimation performance of the VAR specifications.

4.2.1 Motivating the Choice of Stochastic Processes

We choose our shock processes to examine two plausible scenarios in the detection of technology shocks:

1. There are confounding low-frequency but less persistent processes in the data other than

technology;

2. There are business cycle frequency processes in the data.

As described earlier, our NAMS approach has been designed to deal with the first case, while the spectral identifications are targeted at the second. Before we turn to the simulations, we briefly describe the specifications that can generate the two described processes. This includes covering the frequency domain properties of AR(1) and AR(2) processes.

Our choice of driving processes is motivated by the following well-known spectral density factoid. Consider the white noise process ϵ_t, with variance $\gamma(0) = \sigma^2$ and autocovariance function $\gamma(h) = 0$ for $h \neq 0$.

Therefore, the spectrum is

$$\int_{-\pi}^{\pi} S(\omega) d\omega = \gamma(0) = \sigma^2 \tag{27}$$

Now consider the AR(1) process $v_t = \rho v_{t-1} + \epsilon$ with autocovariance function $\gamma(h) = \sigma^2 \rho^{|h|}/(1\text{-}\rho^2)$. The associated spectrum is,

$$S(\omega) = \frac{\sigma^2}{1 - 2\rho \times cos(2\pi\omega) + \rho^2} \tag{28}$$

Notice that when $\rho > 0$ the spectrum is dominated by low-frequency shocks, and in the case of negative autocorrelation, $\rho < 0$, the spectrum is dominated by high frequency components.[13] This simple factoid shows that a specification of a simple AR(1) process for b allows us to generate a confounding low-frequency process.[14] A business cycle frequency shock requires an AR(2) process.

We replicate a sinusoidal business-cycle shock process with a specific frequency (f) using the following AR(2) process:

$$x_t = 2cos(2\pi f)x_{t-1} - x_{t-2}$$

Here we set f such that the shock process for b has a periodicity of 8-quarters (2 years).

[13] To see this, substitute ω for low values (low frequency), say 1/32, or high values, say 1/2.

[14] A negatively-signed AR coefficient would allow us to include a confounding high frequency shock. For a more detailed discussion of the data-generating processes behind a range of spectral densities, see Medel (2014).

In our application, to ensure the cyclical process degrades over time (and avoids a unit root), we multiply both coefficients ($\rho_{b,1}$ and $\rho_{b,2}$) by 0.9. We also calibrate the variance of the shocks to be different for a clear distinction between the driving processes. In both cases, the shock standard deviations are calibrated so that ϵ^z, the technology shock, explains the majority (just over 50%) of the FEVD at period 40. This allows us to demonstrate that even where the shock of interest is the dominant shock, the application of the Max-Share identification will still result in biased estimates.

The two calibrations are shown in Table 3.

Table 3: Shock process calibration for two-variable simulations

Parameter	**Case 1: Low-frequency AR(1)**	**Case 2: Business frequency**
$\rho_{b,1}$	0.3	1.27
$\rho_{b,2}$	0.0	-0.7
σ_b	2	0.7
σ_z	1	1

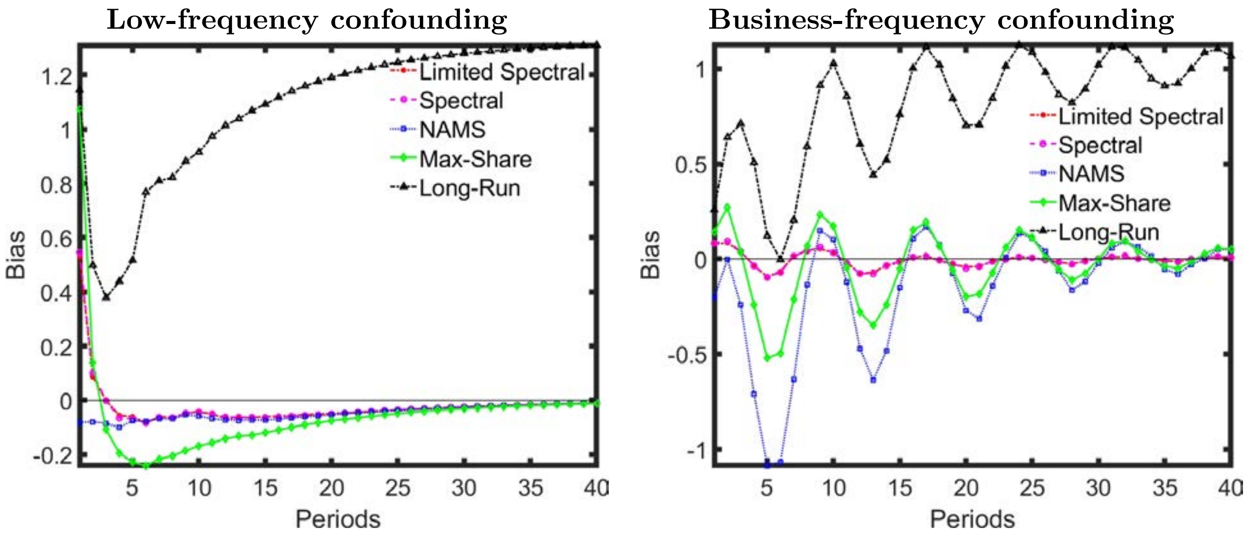
4.2.2 SVAR Performance

Case 1: low-frequency confounding shocks: In the presence of an additional low-frequency, albeit less persistent shock, both the Spectral and NAMS identifications outperform the traditional Max-Share and long-run restriction approaches. The NAMS approach is least affected by the confounding shock, consistent with its intended purpose.

As predicted, the IRF of the Max-Share identification is biased upwards by the higher-variance shock b (Figure 10), even though the targeted shock z explains the majority of the forecast error variance at the 10-year horizon. In the presence of confounding low-frequency shocks (the less persistent AR(1) process), the NAMS approach shows the least IRF bias and shows minimal bias in the estimation of the FEVD share of technology in productivity (Figure 11). This is to be expected: by design, the NAMS approach gives minimal weight to low-persistence processes. The end result is a very high correlation between the estimated NAMS technology shocks and the true underlying shocks (Table 4).

The spectral approaches also show less IRF bias than the Max-Share approach. This might seem unintuitive at first, as the confounding shock is also a low-frequency process, like the targeted technology shock. However, the lower persistence of the variable b relative to z lowers its contribution to the variance at low frequencies, reducing the bias. Referring to equation 28, it is clear that the contribution of the process b to the variance of productivity at low frequencies will be increasing in the size of the persistence parameter ρ.

Figure 10: IRF bias in the presence of low-frequency and business cycle shocks



Note: Absolute bias of technology IRF on the presence of a confounding low, and a confounding business-cycle non-technology shock, respectively.

Case 2: business-frequency confounding shocks: In the presence of the business-cycle frequency confounding shock, the Spectral VAR specifications have a clear advantage. While there is some evidence of contamination in the IRF (Figure 10), it is significantly below the contamination in the Max-Share, NAMS and long-run approaches. The shocks uncovered by the Spectral specifications also have a higher median correlation (0.97) with the true underlying shock than the remaining VARs (Table 4).

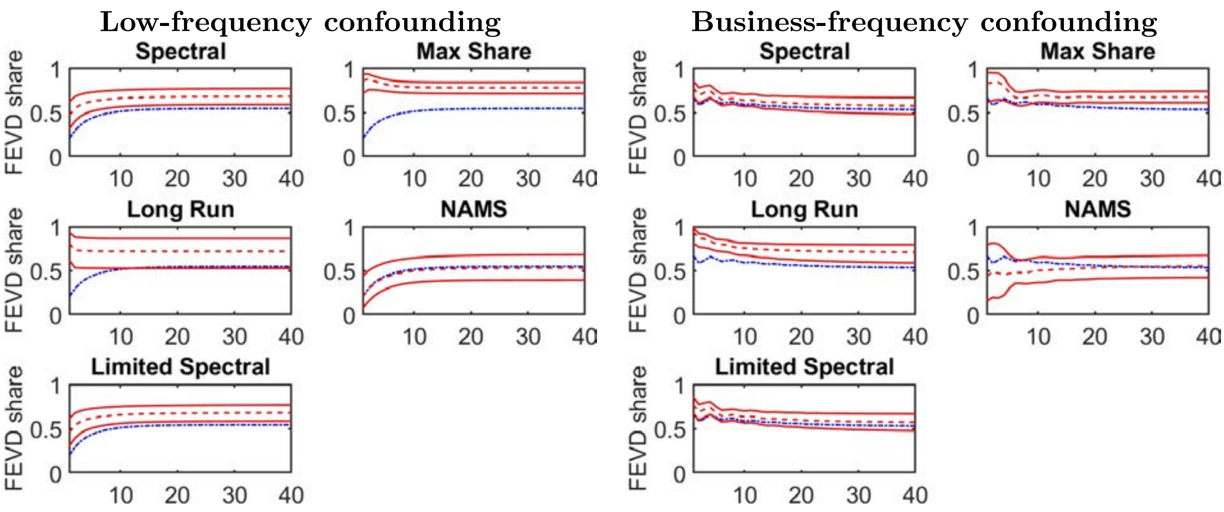
Table 4: Correlation of VAR-identified shocks with true technology shock

Identification	Max-Share	Long-Run	Spectral	Limited Spectral	NAMS
Low-frequency	0.71	0.71	0.92	0.92	0.97
	(0.61, 0.80)	(0.58, 0.84)	(0.84, 0.97)	(0.85, 0.97)	(0.93, 0.99)
Business-frequency	0.71	0.63	0.97	0.98	0.18
	(0.37, 0.92)	(0.54, 0.71)	(0.95, 0.99)	(0.95, 0.99)	(-0.01, 0.71)

Note: 5th and 95th percentiles in brackets.

In addition, the Spectral identifications are less prone to overstate the FEVD share of the technology shock (Figure 11). Both the long-run and Max-Share specifications are prone to overstating the forecast error variance explained by the technology shock, capturing additional variance from the business-cycle shock. In contrast, the NAMS approach initially understates the contribution to the forecast error variance and has larger confidence bands than the spectral approaches.

Figure 11: Estimated and true forecast error variance



Note: Blue= true forecast error variance. Red= estimated median, 16th and 86th percentile error bands

Overall, in the presence of confounding business-cycle frequency shocks, the Spectral approaches have a clear advantage over the traditional Max-Share and Long-Run identification, even where the technology shock dominates the forecast error variance at a standard target horizon (10-years). Appendix 8.3.2 shows the impact of increasing the target horizon to 15 years, finding that the results are robust to this change.

4.3 Summary of Identification Performance

In the case of technology taking the AR(1) form, the Spectral identifications have been shown to perform best in the face of non-technology cyclical shocks, and also show less bias than the Max-Share identification in the face of non-technology low-frequency shocks. However, the NAMS identification most effectively abstracts from non-technology confounding low-frequency shocks. In contrast, business cycle shocks can bias NAMS, in part depending on the amplitude of the cycle of the confounding shock at the target horizon (k). These simple two-variable DGPs also do not account for lag-truncation bias (they do not have an infinite-AR form), which was found to be a source of bias in the DSGE estimations for NAMS.

All identifications, with the exception of long-run restriction, identify technology well when it takes a unit-root form, primarily as it will prove so dominant over confounding shocks. Where technology takes the form of a persistent shock to growth, the spectral identifications show the least bias when estimated on the productivity data in log-differences.

5 Application to US Data

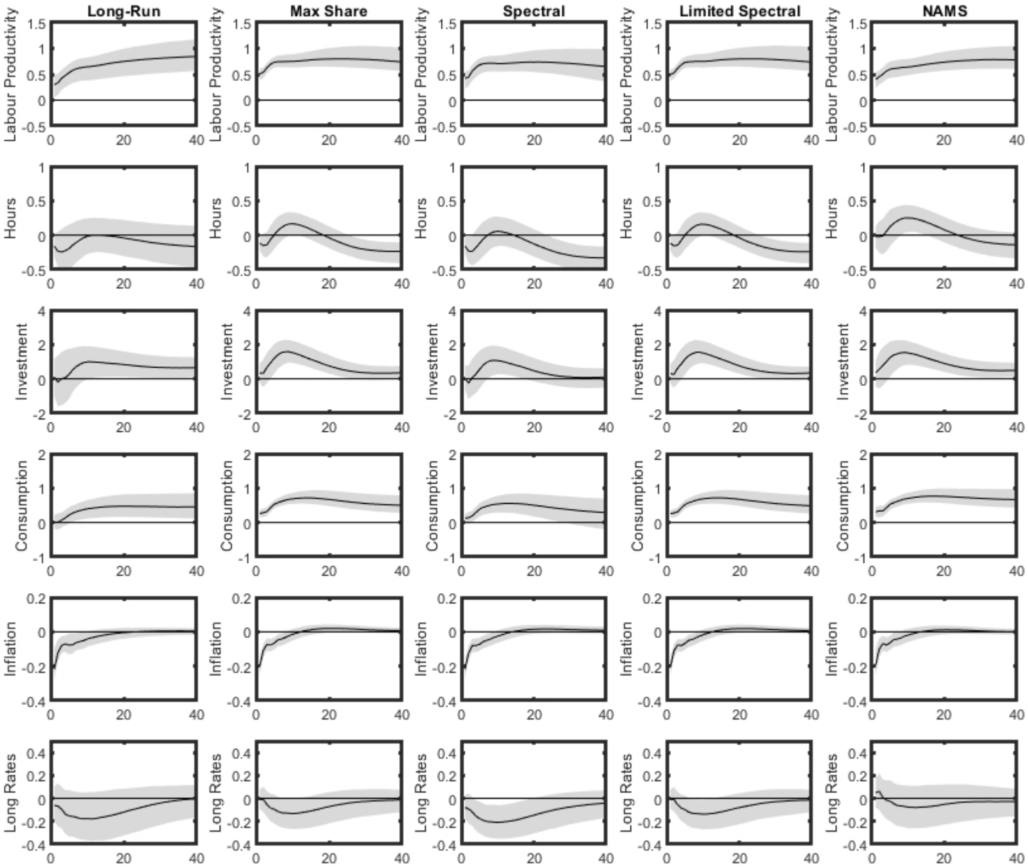
We now apply each identification method actual data to gauge the types of bias that may arise in real world applications. We find that when applied to the US data, each proposed new methodology offers qualitatively similar impulse responses (Figure 12). However, a closer examination of the forecast error variance, the persistence of the IRFs, and the results of estimating the VAR using labor productivity in log-differences are revealing about the likely presence of confounding shocks.

The IRFs and forecast error variance decomposition suggest the presence of lag-truncation bias

We use a 6-variable VAR with 4 lags.[15] The VAR contains logged labor productivity (output per hour), logged total hours worked per capita, the share of investment in total output (including consumer durables and excluding government investment), the share of consumption in total output (excluding consumer durables), PCE inflation, and the yield on the US 10-year treasury. The VAR is estimated between 1953:Q2 and 2018:Q3, with the starting date constrained by the availability of the 10-year treasury yield data (to avoid ZLB issues associated with short-term rates). All data are from the FRED database produced by the Federal Reserve Bank of St. Louis (see Appendix 8.5 for further details).

[15] Our results are robust to alternative lag specifications, given the associated problems highlighted by Canova et al. (2010) and Chari et al. (2008) around the estimation of a process that may have an underlying representation of an AR process with infinite lags. The relative degree of potential lag-truncation bias remains consistent across specifications, as in the CKM exercise.

Figure 12: IRFs from estimation on U.S. data productivity data in levels



Note: 16th and 86th percentile error bands. The long-run identification is estimated using differenced labor productivity and hours worked per capita data. All other identifications use both variables in levels.

While the IRFs provide qualitatively similar results (Figure 12), the forecast error variance decomposition (FEVD) and the initial impact shows important differences between the identifications. The Max-Share and Limited Spectral methods, which have been shown to be the most robust to lag-truncation bias, have higher FEV shares in the initial period than all other methods (Table 5). The Spectral, NAMS, and long-run specification have been shown to be biased downward due to lag-truncation. Another possibility for the higher initial impact of the Max-Share and Limited Spectral methods would be the presence of a confounding low-frequency shock. However, this would not explain why the Spectral and Limited Spectral methods produce different results.

There appears to be few confounding business-cycle frequency shocks in the productivity levels data. This can be seen by the similarities between the Max-Share and Limited Spectral FEVD identifications. One way of thinking about the relationship between the two methodologies is that the limited spectral approach is simply a Fourier transform of the Max-Share identification, with the additional feature of excluding certain frequencies. The forecast error variance matrix with which the maximization takes place for each can be written as:

$$V_{\tau}^{Max-Share} = \sum_{\tau=0}^{k-1} D^{\tau} \Sigma_u D^{\tau\prime} \tag{29}$$

and

$$V_{\tau}^{L.Spectral} = \sum_{\tau=0}^{k-1} D^{\tau} \left(e^{i\tau\omega}\right) \Sigma_u D^{\tau} \left(e^{i\tau\omega}\right)^{\prime} \tag{30}$$

Table 5: Proportion of labor productivity FEVD explained at horizon t in each identification

Identification	Max-Share	Long-Run	Spectral	Limited Spectral	NAMS
Levels: t=1	0.45	NA	0.31	0.44	0.29
Levels: t=20	0.89	NA	0.81	0.89	0.69
Levels: t=40	0.93	NA	0.84	0.92	0.83
Differences: t=1	0.99	0.16	0.36	0.35	0.13
Differences: t=20	0.40	0.56	0.92	0.92	0.11
Differences: t=40	0.32	0.64	0.94	0.94	0.13

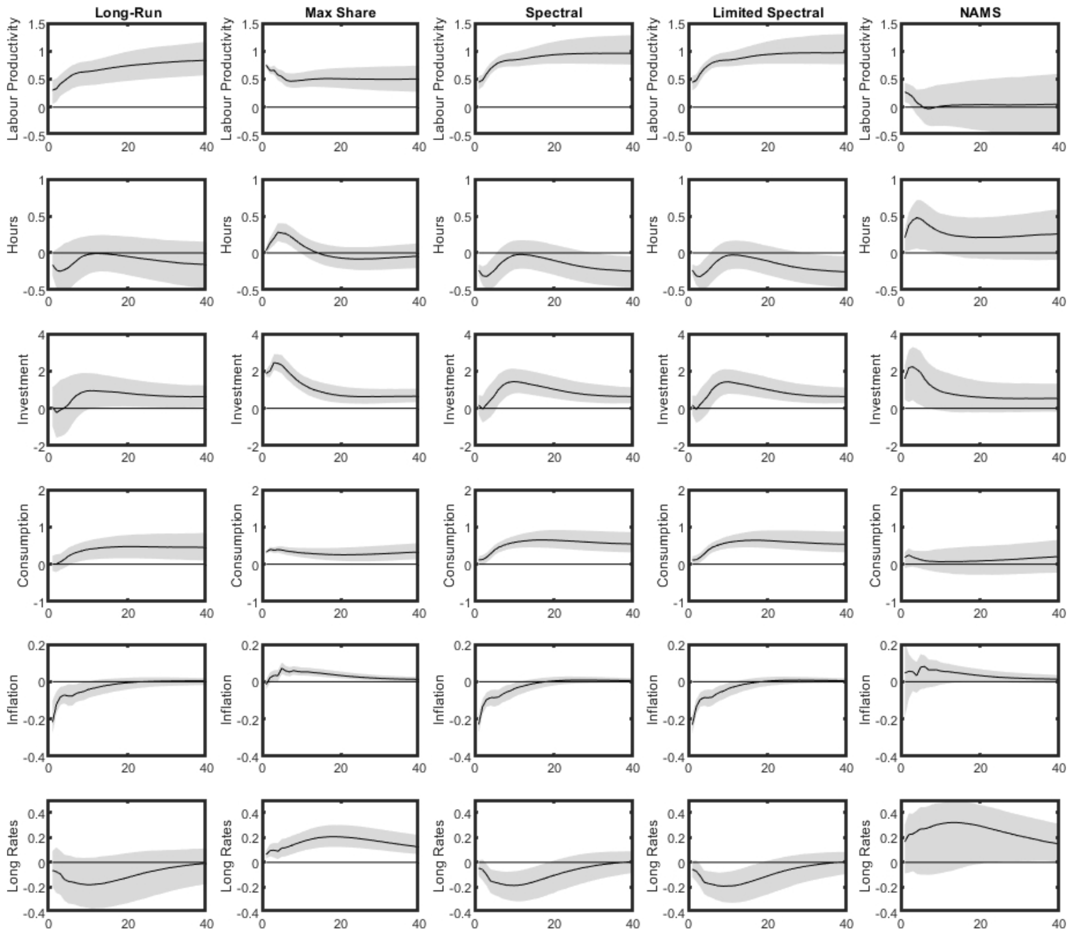
Therefore, medium and high-frequency volatility must be a sufficiently small proportion of the variance at this horizon so that the maximization problems are essentially equivalent. In contrast, the growth rate of US productivity exhibits a wide range of frequencies driving its spectral density, unlike the level, suggesting an estimation of the VAR with productivity in log-differences will also be informative.

Spectral identifications best capture technology shocks with persistent effects on productivity growth

When estimating the VARs in differences (Figure 13), the Spectral approaches show a more persistent IRF than when estimated on the level of productivity, and a similar share of forecast error variance explained of productivity to one another. This demonstrates similar circumstances to the model-based scenario with simulated data in the case of technology taking the form of a unit-root plus stochastic growth process (see Section 4.1). Furthermore, the Max-Share and

NAMS approaches appear to be significantly confounded by non-technology influences. The differenced spectral VARs are therefore likely producing the least biased IRFs for the response of productivity to technology.

Figure 13: IRFs from estimation on labor productivity log-differences



Note: 16th and 86th percentile error bands

Only the spectral identifications produce consistent estimates when estimating on data in levels and differences

Comparing the impulse responses across the two specifications (productivity estimated in levels versus differences), shows that only the spectral estimators produce consistent impulse responses in each case (the long-run identification is always estimated in differences). In response to a positive technology shock, productivity, consumption, and investment rise, while hours, inflation, and the long rate fall initially. The confidence bands around the spectral estimates are also tighter than those produced by the other methods. However, while the Max-Share impulses are mainly consistent across levels and differences, there are disparities in the impulse of hours and long rates: both hours and the long rates are negative when productivity enters the VAR in log-levels and positive when estimated in differences. These results alongside the less persistent productivity IRF suggest that non-technology factors, such as a New Keynesian demand shock, have confounded the identified shock.

NAMS produces contradictory or non-informative impulse responses across all variables except for consumption and investment when estimated on labor productivity in differences. Additionally, the accompanying wide confidence bands of the difference specification precludes us from drawing any concrete conclusions and again manifest themselves in the very low explained FEVD (Table 5). This is to be expected, given the impact of a technology shock on productivity *growth* after 10 years is understandably small and uncertain. Simulations in the unit-root case also showed a weaker performance from NAMS relative to the spectral methods.

The Limited Spectral methodology is the most robust across specifications

In summary, the US data looks to give rise to lag-truncation bias and contain technology shocks with persistent effects on productivity growth. The Limited Spectral estimator can reliably handle this data generating process in both levels and differences for productivity. The NAMS and Spectral approaches are less able to cope with the presence of lag-truncation bias when productivity in log-levels is used in the VAR, although the Spectral approach works well in log-differences, where lag-truncation bias is a smaller issue.

6 Detecting Cyclical Shocks: dissecting the business-cycle anatomy

The Max-Share and the Spectral VAR methodologies have been recently applied to finding dominant business-cycle frequency shocks. For example, Angeletos and Dellas (2020) find a single primary driver of multiple macroeconomic variables at business cycle frequencies. Levchenko and Pandalai-Nayar (2018) use the Max-Share methodology to identify the impact of changes in economic sentiment on US output.

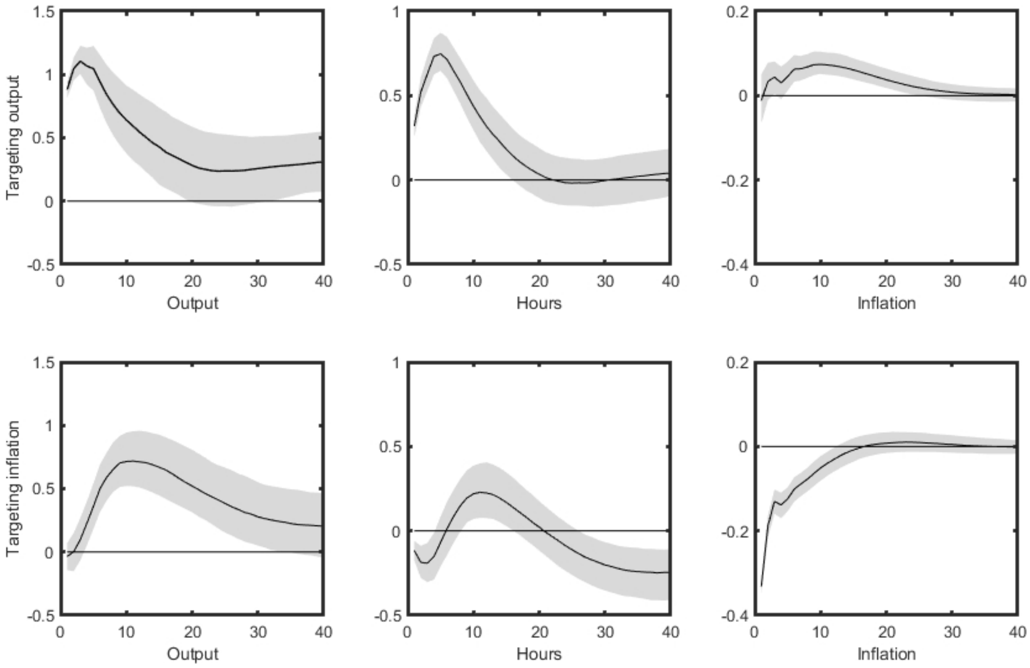
We note that the same logic of confounding shocks will also apply in these cases. Most notably, using the Max-Share and Spectral identifications to find the shock that maximizes the forecast error variance or variance within a particular frequency band does not necessarily identify a single structural shock. For example, Giannone et al. (2019), find a shock which maximizes the variance of unemployment at business-cycle frequencies but note that this statistical-derived identification will encompass a linear combination of structural shocks. We formalize this statement in this paper, finding that the shock will encompass a combination of structural shocks broadly in proportion to their relative contribution to the variance at that horizon or frequency band. We propose that the same logic outlined in Appendix section 8.1 would also hold in the case of targeting the shock which maximizes the variance within a business-cycle frequency band. Take for example two structural shock drivers affecting 2 endogenous variables at business-cycle frequencies A^{BC-1} and A^{BC-2}, where A^{BC-1} drives the majority of the variance at these frequencies. In the case where only these two drivers existed, column one of the identified rotation matrix would weight the two shocks according to their contribution to the variance at the desired frequency band ω and horizon k (in the Limited Spectral this would be 40 quarters and ∞ in the standard Spectral approach). It would not simply 'pick out' the dominant shock.

$$\tilde{q}(k) = \begin{bmatrix} \frac{A_{11}^{BC-1} + \sum_{\tau=1}^{k} (D_{\tau}^{11}\left(e^{i\tau\omega}\right) A_{11}^{BC-1} + D_{\tau}^{12}\left(e^{i\tau\omega}\right) A_{21}^{BC-1})}{A_{12}^{BC-2} + \sum_{l=1}^{k} (D_{\tau}^{11}(e^{i\tau\omega}) A_{12}^{BC-2} + D_{\tau}^{12}(e^{i\tau\omega}) A_{22}^{BC-2})} \\ 1 \end{bmatrix}$$

This will make it difficult to interpret IRFs of the identified shock in question, given they will represent a combination of drivers with only an *a priori* understanding of their relative weights.

Furthermore, the variance of many macroeconomic drivers in the business-cycle frequency domain are more prone to contamination than in the low-frequency range. For example, at business-cycle frequencies a wide range of shocks influence real and nominal variables. The competing effects of supply-side and demand-side influences may therefore be difficult to disentangle. Using the Spectral methodology, Angeletos and Dellas (2020) argue that dominant business-cycle drivers of real variables such as GDP, unemployment, and investment have very little impact on inflation, therefore ruling out inflationary demand shocks of the standard New Keynesian variety as key drivers of the business cycle. Using the same VARs used to evaluate the identification of technology in the US data, this finding can be confirmed. By targeting the shock the maximizes the variance of GDP at business cycle frequencies (6-32 quarters), a shock which boosts output by one percent in the short-run is found, but which results in inflation rising by just 0.05 percentage points at its peak. In contrast, the shock that maximizes the variance of inflation at business cycle frequencies has a more substantial impact on inflation (-0.3 percentage points) but a smaller impact on output.

Figure 14: Targeting Output and Inflation at Business-cycle Frequencies



Note: 16th and 86th percentile error bands. The first row shows impulse responses to the shock which maximises the variance of output at business-cycle frequencies (6-32 quarters). The second targets inflation. The VAR consists of US data on labor productivity, hours-worked, the investment to GDP ratio, the consumption to GDP ratio, the consumption expenditure deflator (inflation), and the 10-year US treasury yield.

We argue that an alternative candidate explanation for the absence of a significant inflationary impact from the main driver of output at business cycle frequencies is the presence of competing demand and supply-side influences, which dull the response of inflation. Demand-side drivers of output will cause inflation to rise, while supply-side drivers of output can cause output to rise while reducing inflation (technology and markup shocks for example). The presence of both types of shock in driving output will result in a hybrid IRF to inflation, consisting of a mix of the effects of both shocks.

A simple New Keynesian model demonstrates how the Spectral methodology can produce these results even in the case where inflationary demand shocks drive the majority of the variance of output:

$$y_t = \frac{-1}{\sigma}\left(R_t - E_t[\pi_{t+1}]\right) + E_t[y_{t+1}] + \eta_t \tag{31}$$

$$\pi_t = \kappa MC_t + \beta E_t[\pi_{t+1}] \tag{32}$$

$$MC_t = (\sigma + \chi)y_t - (1 + \chi)a_t \tag{33}$$

$$R_t = (\phi_t)y_t + \phi_\pi \pi_t \tag{34}$$

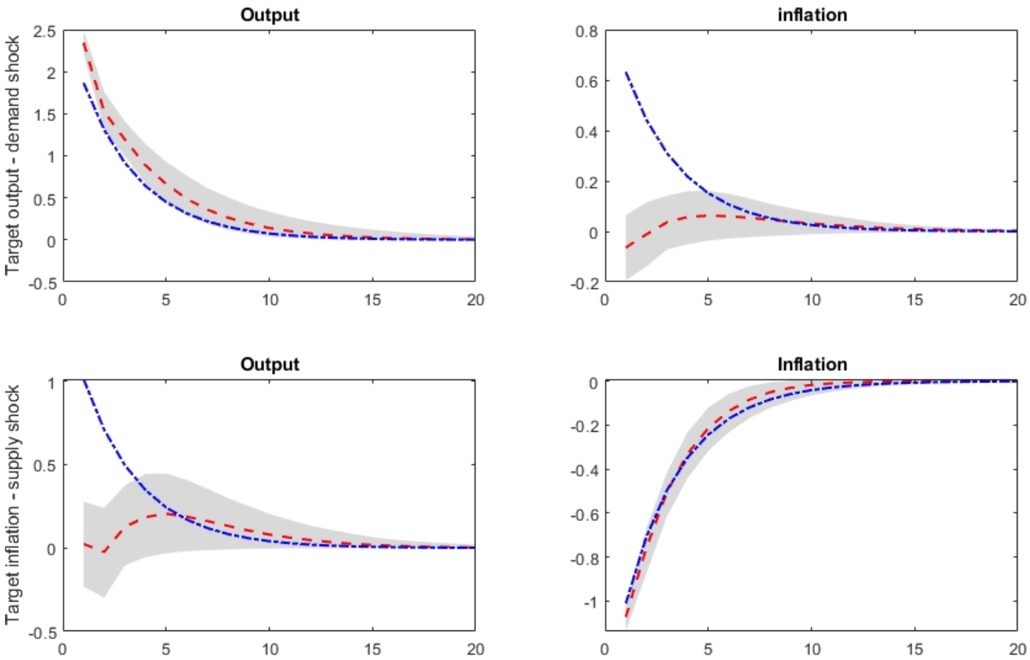
Where y_t is output, R_t is the nominal interest rate, π_t is inflation, MC is marginal costs. η_t represents a demand (preferences) shock, while a_t reflects a supply-side technology shock. σ is the inter-temporal elasticity of substitution, χ is the Frisch elasticity of labor supply. κ is the slope of the Phillips curve, and is a function of the probability of not being able to reset prices each period (θ) and the discount rate (β) :$\kappa = (1 - \theta)(1 - \beta\theta)/\theta$.

In this standard framework, there are two key drivers of fluctuations in output and inflation. Let us suppose that these are transitory drivers, with sizable contributions to the business-cycle. These shocks therefore follow auto-regressive processes with a coefficient of 0.7 on the shock's auto-regressive coefficients ($a_t = 0.7a_{t-1} + \epsilon_a$ and $\eta_t = 0.7\eta_{t-1} + \epsilon_\eta$). Elasticities are set to 1 for simplicity (σ and χ) while the inflation aversion coefficient is set to 1.5 and the output gap coefficient is set to 0.5 in the policy-rate equation. Shock variances are set at $\sigma_a^2 = 0.004$ and $\sigma_\eta^2 = 0.002$, which results in the supply shock explaining roughly three quarters of the variance of inflation, and the demand shock explaining roughly three-quarters of the variance of output.

Simulating the data for 250 periods and then separately identifying the dominant business-cycle driver of output and inflation in a two variable VAR using the Spectral methodology produces the postulated result. Despite the presence of clearly separate driving processes, the IRFs for inflation end up as a mixture between the supply and demand shock IRFs. This provides the appearance of a subdued response of inflation to a demand shock, despite the demand shock dominating the variance of output in the true DGP. In addition, the response of output to the supply shock is subdued due to the offsetting impact of the demand shock.

Both the Spectral and Limited Spectral methodologies will erroneously conclude that a dominant structural demand shock has little effect on inflation. Practitioners will need to have an *a priori* understanding of the range of potential confounding shocks in these exercises, and interpret the results accordingly.

Figure 15: Targeting Output and Inflation at Business-cycle Frequencies in a two shock New Keynesian model



Note: 16th and 86th percentile error bands. The first row shows impulse responses to the shock which maximises the variance of output produced by the simple New Keynesian model at business-cycle frequencies (6-32 quarters). The second targets inflation. Blue lines indicate the true IRF of the demand shock (top row) and supply shock (bottom row). Shared area and red line indicate the VAR-estimated IRF median and 68% confidence bands.

7 Conclusion

This paper documents the biases that can be introduced into the long-run and variance-maximising methodologies by confounding shocks. We show theoretically why this is the case, and why Monte Carlo DSGE simulations have previously failed to account for this issue given the overwhelming dominance of technology shocks in many such models. Three alternative SVAR identifications are proposed to deal with confounding shocks with different frequency domain properties, two of which are new (Limited Spectral and NAMS).

All proposed approaches and the Max-Share methodology are found to be more robust to multiple sources of bias compared to the long-run identification of technology shocks. Using a simple two-variable model, we show that the NAMS approach is less susceptible to confounding low-frequency shocks when trying to identify highly persistent technology shocks. However, it suffers from lag-truncation bias to a higher degree than the Max Share and Spectral approaches and is unable to capture technology shocks when estimated on differenced labor productivity data unless the technology shock has highly persistent effects on labor productivity *growth*. Two types of SVAR identifications in the frequency domain show significantly reduced estimation bias in the presence of business-cycle frequency confounding shocks relative to technology. The newly developed Limited Spectral identification also reduces lag-truncation bias compared to existing spectral methodologies to a degree. Table 6 summarizes the relative performance of each method under the gamut of DGPs examined, showing the top three performing methods in each. Notice that both spectral methods are top-ranked in all the DGPs considered in this paper. Therefore, in the absence of clear information about the appropriateness of one particular method, a spectral approach appears a safe one to take. Additionally, given small sample and lag-truncation estimation issues, the Limited Spectral method is our preferred specification.

We demonstrate that the US productivity data suggests the presence of persistent effects of technology on labor productivity growth. The Spectral identifications are the only methods able to both robustly estimate the SVAR using labor productivity in log-differenced form, which is better able to capture this feature of the data. The Max Share approach, which otherwise compares well with the Spectral methodologies, captures a much less persistent impulse response when estimated in this differenced form, reflecting the presence of confounding non-technology shocks.

Finally, we illustrate how our findings concerning confounding shocks also extends to the identification of dominant business-cycle shocks. In particular, we demonstrate how misleading IRFs can be uncovered when identifying dominant business-cycle shocks in the US data using data generated by a simple New Keynesian model.

Table 6: Performance of identifications in different simulations

	Identification	Long-Run	Max-Share	Spectral	Limited Spectral	NAMS
Levels	**New Keynesian**		+	+	+	-
	RBC		+	+	+	-
Differences	**New Keynesian**	-	+	+	+	-
	RBC	-	+	+	+	-
Levels	**CKM impact bias**	-	+	+	+	-
Differences	**CKM impact bias**	+	+	+	+	-
Levels	**CKM lag-truncation bias**	-	+	+	+	-
Levels	**2-variable unit root**		+	+	+	+
Differences	**2-variable unit root**	-	-	+	+	+
Business-cycle confounding	**2-variable stationary**	-	-	+	+	-
Low-freq confounding	**2-variable stationary**	-	-	+	+	+

Note: Top three specifications are designated with "+", except in cases where an additional specification has a similar performance (long-run in the differenced CKM impact-bias, or 2-variable unit root levels where 4 identifications have near-idential performance), or in cases where the third best specification is significantly behind the top 2 (business-frequency confounding shocks).

8 Appendix

8.1 What shock is Max-Share capturing?: Sources of bias

8.1.1 Low and high-frequency drivers of forecast errors

In this Appendix section, we formally demonstrate that the Max-Share identification is prone to contamination from shocks of higher-frequency or lower persistence than desired. In the below, we show the contamination from a high frequency shock when the econometrician attempts to identify a low-frequency shock. However, the logic below could easily be replaced by a high-persistence shock contaminated by a low persistence shock to demonstrate the same result.

A series Y is driven by two structural shocks, a low-frequency shock ϵ^l and a high frequency shock ϵ^h with $cov(\epsilon^l, \epsilon^h) = 0$. The forecast error at horizon k is a function of the structural impulse responses at each horizon $A_t = [A_t^l \, A_l^h]$. The series A_t is formed of the reduced-form MA coefficients and the identification matrix, $A(L) = B(L)A_0$.

$$F(Y)_k = \sum_{i=0}^{i=k} \begin{bmatrix} A_i^l & A_i^h \end{bmatrix} \begin{bmatrix} \epsilon_i^l \\ \epsilon_i^h \end{bmatrix}$$

In turn, the forecast error variance of Y is:

$$F^2(Y)_k = \sum_{i=0}^{i=k} (A_i^l \epsilon_i^l)^2 + \sum_{i=0}^{i=k} (A_i^h \epsilon_i^h)^2$$

The proportion of $F^2(Y)_k$ explained by low-frequency shocks is increasing in the persistence of A_l relative to A_h as k increases. It is also increasing in the relative variance of the impact of the low frequency shock $\left(\frac{A_0^l \epsilon^l}{A_0^h \epsilon^h}\right)^2$. For the researcher looking to isolate the shock which dominates the low-frequency dynamics of a particular series, k must be set sufficiently high for the low-frequency shock to dominate the forecast error variance.

Lemma 2 *Where a series is driven by a combination of low and high frequency processes, the low frequency shock will only account for the majority of the forecast error variance for sufficiently large k, such that:*

$$\frac{\sum_{i=0}^{i=k} A_i^l}{\sum_{i=0}^{i=k} A_i^h} > \frac{A_0^h \epsilon^h}{A_0^l \epsilon^l}$$

By definition, the series of high frequency shock coefficients will be declining at a faster rate than for low frequency coefficients. In some cases, the variance of the low frequency shock will exceed the variance of the high frequency shock, and dominate the forecast error variance from the initial period.

The probability of mistaking a transitory shock for a persistent one will be low where the low-frequency shock takes the form of a unit root process. Take a limiting case, where $Y_t = \sum_{i=0}^{T} A_i^l \epsilon_i^l + A_0^h \epsilon_t^h$, ie. Y_t is driven by a permanent I(1) shock and a transitory white noise process. Here, $A_{0:k}^l = 1$, while $A_0^h = 1$, $A_{1:k}^h = 0$. The low frequency shock will dominate the forecast error variance for $k \geq \left(\frac{A_0^h \epsilon^h}{A_0^l \epsilon^l}\right)^2$. But for other processes, there may be confounding shocks with similar frequencies to the shock of interest. For low frequency shocks with less than infinite periodicities, and in the presence of persistent business cycle shocks, the researcher is liable to identify a shock other than the low-frequency shock without sufficiently high k.

8.1.2 Solving the maximization problem

Even where the low-frequency shock dominates the forecast error at horizon k, the shock captured by Max-Share will not consist only of this underlying shock. Here we show that the shock which maximizes the contribution to the forecast error variance of productivity is a combination of the low and high-frequency structural shock, rather than the dominant underlying structural shock.

Start with the definition

$$Y_t = B(L)u_t$$

with the true underlying structural shock $A_0^{-1}u = \epsilon$, where ϵ refers to the low and high frequency shocks described in the previous example. In the Max Share approach, the search for A_0 begins with $\tilde{A}_0$, a Cholesky decomposition of Σ_u and the impulse matrix $\tilde{A}^\tau = D^\tau \tilde{A}_0$ for the impact of the shock at horizon τ. The 'true' structural shock is defined using an unknown orthonormal matrix Q, such that the true structural impulse matrix $A^\tau = \tilde{A}^\tau Q$ and the forecast error at horizon k is

$$F(Y)_k = \sum_{\tau=0}^{k} \tilde{A}^\tau Q \epsilon_{t+k-l} = \sum_{\tau=0}^{k} A^\tau \epsilon_{t+k-\tau}$$

with the error variance also equivalent for both the true shock and the initially proposed Cholesky decomposition form since:

$$\Sigma(k) = \sum_{\tau=0}^{k} \tilde{A}^\tau \tilde{A}^{\tau\prime} = \sum_{\tau=0}^{2} \tilde{A}^\tau Q Q' \tilde{A}^{\tau\prime}$$

As in Uhlig (2003), isolating the shock which explains the largest proportion of the forecast error variance reduces to an eigenvector decomposition problem. This means searching for an orthonormal vector q (of the matrix Q) which maximizes the forecast error variance for variable ii.

$$\begin{aligned} F^2(Y)_k &= \sum_{\tau=0}^{k} \Big((D^\tau \tilde{A}_0 q)(D^\tau \tilde{A}_0 q)' \Big)_{ii} \\ &= q' S(k) q \end{aligned}$$

Where

$$S(k) = \sum_{l=0}^{k} (D_i^{\tau} \tilde{A}_0)(D_i^{\tau} \tilde{A}_0)'$$

Maximizing $q'S(k)q$ is a Lagrangian with the constraint that $q'q = 1$

$$L(q) = q'S(k)q - \lambda(q'q - 1)$$

whose first order conditions reduce to solving for the eigenvector associated with the largest eigenvalue of $S(k)$

$$S(k)q = \lambda q$$

.

Will this q be equivalent to the first column of Q defining the true low and high-frequency structural shocks, $A^0 = \tilde{A}^0 Q$?

We will demonstrate that, in an attempt to isolate the low-frequency shock, the q in the Max-Share rotation will still contain features of the high-frequency shock, the extent depending on the forecast horizon, the relative variances of the shocks, and the relative persistence of the shocks. While we demonstrate this for the simple 2x2 case we argue that the results hold for higher dimension VARs.

As in the previous example, the true underlying low and high-frequency shocks can be written as

$$A_0 \epsilon = \begin{bmatrix} A_{11}^l & A_{12}^h \\ A_{21}^l & A_{22}^h \end{bmatrix} \begin{bmatrix} \epsilon^l \\ \epsilon^h \end{bmatrix}$$

Here, A_{11}^l and A_{21}^l refer to the impacts of the low frequency shock on variables 1 and 2 respectively in period 0, while A_{22}^h and A_{12}^h refer to the impacts of the high frequency shock on variable 2 and 1 respectively.

The total forecast error variance is equivalent when using the structural form of the shocks and the reduced form since

$$\Sigma_\mu = A_0 A_0'$$

The forecast error variance can be computed using the structural shocks and the reduced form MA-representation coefficients $D^{i:T}_{2\times 2}$. We are only interested in the impacts on the first endogenous variable of the system (for example productivity), and therefore only require the first row of B for our computation of the forecast error variance.

$$FEV(K) = \sum_{i\tau=0}^{k} \left(\begin{bmatrix} D^{11}_{\tau} D^{12}_{\tau} \end{bmatrix} \begin{bmatrix} A^{l}_{11} & A^{h}_{12} \\ A^{l}_{21} & A^{h}_{22} \end{bmatrix} \begin{bmatrix} \epsilon^{l} \\ \epsilon^{h} \end{bmatrix} \right)' \left(\begin{bmatrix} D^{11}_{\tau} D^{12}_{\tau} \end{bmatrix} \begin{bmatrix} A^{l}_{11} & A^{h}_{12} \\ A^{l}_{21} & A^{h}_{22} \end{bmatrix} \begin{bmatrix} \epsilon^{l} \\ \epsilon^{h} \end{bmatrix} \right)$$

At $K = 0$, $[D^{11}_0 \, D^{12}_0] = [1\,0]$ since the second variable will not have a contemporaneous impact on the first.

Therefore:

$$S(0) = F^2(Y)_0 = \begin{bmatrix} (A^{l}_{11})^2 & A^{l}_{11}A^{h}_{12} \\ A^{h}_{12}A^{l}_{11} & (A^{h}_{12})^2 \end{bmatrix}$$

As in Uhlig, finding the shock that maximizes the forecast error variance of variable i reduces to an eigenvector problem.

Returning to the Lagrangian where we maximize $q'S(k)q$ subject to $q'q = 1$. In the case of $S(0) = FEV(0)$, the eigenvalue problem is simple. Here, there are two eigenvalues found, $\lambda = [(A^2_{11} + A^2_{12}), 0]$. The eigenvector associated with the largest eigenvalue, the non-zero value, is simply a ratio of the impact of the two shocks on variable 1 at time zero. Normalizing the second contribution to 1, the un-normalized eigenvector can be written as:

$$\tilde{q}(0) = \begin{bmatrix} \frac{A^{l}_{11}}{A^{h}_{12}} \\ 1 \end{bmatrix}$$

The normalized eigenvector q can be obtained by dividing each element of $\tilde{q}$ with its Euclidean length. This also demonstrates the generalized solution to the eigenvector problem for a symmetric 2x2 matrix as a function of the square root of the diagonal elements of S. $\tilde{q}$ will always be proportional to the standard deviation of variable 1 driven by each shock. As this will vary over time, the identified shock will also vary over the timespan used to calculate the forecast error variance.

In period 1, $D = [D^{11}_1 \, D^{12}_1]$, and therefore:

$$\mathrm{S}(1) = \mathrm{S}(0) + \left(\begin{bmatrix} D_1^{11} D_1^{12} \end{bmatrix} \begin{bmatrix} A_{11}^l & A_{12}^h \\ A_{21}^l & A_{22}^h \end{bmatrix} \begin{bmatrix} \epsilon^l \\ \epsilon^h \end{bmatrix} \right)' \left(\begin{bmatrix} D_1^{11} D_1^{12} \end{bmatrix} \begin{bmatrix} A_{11}^l & A_{12}^h \\ A_{21}^l & A_{22}^h \end{bmatrix} \begin{bmatrix} \epsilon^l \\ \epsilon^h \end{bmatrix} \right)$$

$$= \begin{bmatrix} (A_{11}^l)^2 + (D_1^{11} A_{11}^l + D_1^{12} A_{21}^l)^2 & A_{11}^l A_{12}^h + (D_1^{11} A_{11}^l + D_1^{12} A_{21}^l)(D_1^{11} A_{12}^h + D_1^{12} A_{22}^h) \\ A_{12}^h A_{11}^l + (D_1^{11} A_{11}^l + D_1^{12} A_{21}^l)(D_1^{11} A_{12}^h + D_1^{12} A_{22}^h) & (A_{12}^h)^2 + (D_1^{11} A_{12}^h + D_1^{12} A_{22}^h)^2 \end{bmatrix}$$

The eigenvector is now a function of the ratio of the impact of shock 1 on variable 1 in periods 1 and 2 relative to the impact of the high frequency shock in both periods:

$$\tilde{q}(1) = \begin{bmatrix} \frac{A_{11}^l + (D_1^{11} A_{11}^l + D_1^{12} A_{21}^l)}{A_{12}^h + (D_1^{11} A_{12}^h + D_1^{12} A_{22}^h)} \\ 1 \end{bmatrix}$$

We can then generalize the form of the eigenvector q as a function of the forecast error variance horizon chosen:

$$\tilde{q}(k) = \begin{bmatrix} \frac{A_{11}^l + \sum_{\tau=1}^k (D_\tau^{11} A_{11}^l + D_\tau^{12} A_{21}^l)}{A_{12}^h + \sum_{l=1}^k (D_\tau^{11} A_{12}^h + D_\tau^{12} A_{22}^h)} \\ 1 \end{bmatrix}$$

By definition, the ratio $\frac{\sum_{\tau=1}^k (D_\tau^{11} A_{11}^l + D_\tau^{12} A_{21}^l)}{\sum_{\tau=1}^k (D_\tau^{11} A_{12}^h + D_\tau^{12} A_{22}^h)}$, will be increasing over time, as shock 1 has been designated to be more persistent.

As k increases, the ratio of $\frac{\tilde{q}_1}{\tilde{q}_2}$ in $\tilde{q} = \begin{bmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{bmatrix}$ will increase, placing more weight on the persistent shock. In all cases however, the shock found will be a linear combination of the persistent and non-persistent shocks. The ratio will also depend on the initial variance of the respective shocks $(\frac{A_{11}^l}{A_{12}^h})$, and their relative persistence: $\frac{\sum_{\tau=1}^k (D_\tau^{11} A_{11}^l + D_\tau^{12} A_{21}^l)}{\sum_{\tau=1}^k (D_\tau^{11} A_{12}^h + D_\tau^{12} A_{22}^h)}$.

8.2 DSGE Model Specifications

New Keynesian model. The New Keynesian model used is that of (Erceg et al., 2005). The consumer utility function includes habit persistence and disutility of labor:

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \log \left(C_{t+j} - \phi_c \bar{C}_{t+j-1} \right) - \chi_{0,t+j} \frac{N_{t+j}^{1+\chi}}{1+\chi} \right\}$$

The persistence parameter is set close to the mean value in Smets and Wouters (2007) ($\phi_c = 0.6$).

τ_{Nt} and $\chi_{0,t}$ are a labor tax rate shock and a labor supply shock, respectively. Both labor market innovations are assumed to follow AR (1) processes,

$$\tau_{Nt} = \rho_N \tau_{Nt-1} + \sigma_{\tau_N} \epsilon_{\tau_N t}$$

$$\chi_t = \rho_\chi \chi_{t-1} + \sigma_\chi \epsilon_{\chi t}$$

Households face a budget constraint such that consumption, investment, and bond purchases must equal after-tax labor and capital income.

$$C_t + I_t + \frac{1}{1+r_t} B_{t+1} - B_t = (1-\tau_{Nt}) W_t N_t + T_t + \Gamma_t + R_{K_t} v_t K_t +_t -0.5 \phi_K K_t (\frac{I_t}{K_t} - \hat{\delta})^2$$

Variable capacity utilization is incorporated into the production function so that variation in the Solow residual reflects both changes in technology and movements in the unobserved level of capacity utilization $(u_t,)$. The production function is given by:

$$Y_t = (u_t K_t)^\theta \left((Z_t V_t) N_t \right)^{1-\theta}$$

Technology, Z_t, evolves as a unit root process according to:

$$log(Z_t) - log(Z_{t-1}) = \mu_z + \sigma_Z \epsilon_{zt}$$

Intermediate goods are produced by monopolistically competitive firms with a Calvo pricing structure, resulting in a Phillips curve with the following structure:

$$\pi_t = \beta \pi_{t+1} + \kappa_p \left[\zeta_t - (y_t - n_t) \right]$$

where the slope is determined by the probability of prices resetting (ψ_p)

$$\kappa_p = \frac{1-\psi_p}{\psi_p} (1 - \psi_p \beta)$$

Households supply monopolistically differentiated labor services to a competitive "labor market aggregator". The inertia in wage adjustment results in a wage Phillips curve of the form:

$$\pi_t^\omega = \beta E_t \pi_{t+1}^\omega + \kappa_w \left(\chi n_t + \chi_t - \lambda_{ct} - \zeta_t + \frac{1}{1-\tau_N} \tau_{Nt} \right)$$

where

$$\kappa_w = \frac{1-\psi_w}{\psi_w \left(1+\chi \frac{1+\theta_w}{\theta_w}\right)} (1 - \psi_w \beta)$$

and Changes in real wage, ζ, are related to nominal wage and price inflation as follows:

$$\Delta\zeta_t = \pi_t^\omega - \pi_t - \sigma_z \epsilon_{zt}$$

The economy satisfies the following resource constraint

$$\left(1 - \frac{g}{y}\right) y_t = \frac{c}{y} c_t + \frac{i}{y} i_t + g_t$$

where $c/y = (1 - i/y - g/y)$ denotes the steady state consumption share of GDP. g_t is a government spending shock that follows an AR process:

$$g_t = \rho_g g_{t-1} + \sigma_g \epsilon_{gt}$$

Finally, the model is closed with an interest rate rule describing monetary policy:

$$i_t = \gamma_i i_{t-1} + \gamma_\pi \pi_t^{(4)} + \gamma_y \Delta y_t^{(4)} + \sigma_f \epsilon_{ft}$$

where interest rate is set in response to year-on-year inflation and output growth and ϵ_{ft} is a monetary policy shock.

CKM 2008 RBC model

Consumers' utility functions are given by

$$E_o \sum_{t=0}^{\infty} [\beta(1+\gamma)]^t U(c_t, l_t))]$$

where c is consumption, l is per capita labor, β is the discount rate and γ the population growth rate. Consumers maximize utility subject to the budget constraint:

$$c_t + (1+\tau_x)[(1+\gamma)k_{t+1} - (1-\delta)k_t] = (1-\tau_{lt})w_t l_t + r_t k_t + T_t$$

Where τ_x is a tax on investment, k is the capital stock, δ the depreciation rate, w wages, r the rental rate on capital and T a lump sum transfer.

Firms face a resource constraint:

$$c_t + (1+\gamma)k_{t+1} = y_t + (1-\delta)k_t$$

and face a standard constant returns to scale production function:

$$Y_t = k_t^{\theta}(Z_t l_t)^{1-\theta}$$

The technology and non-technology shocks (a stochastic tax on labor income) evolve according to

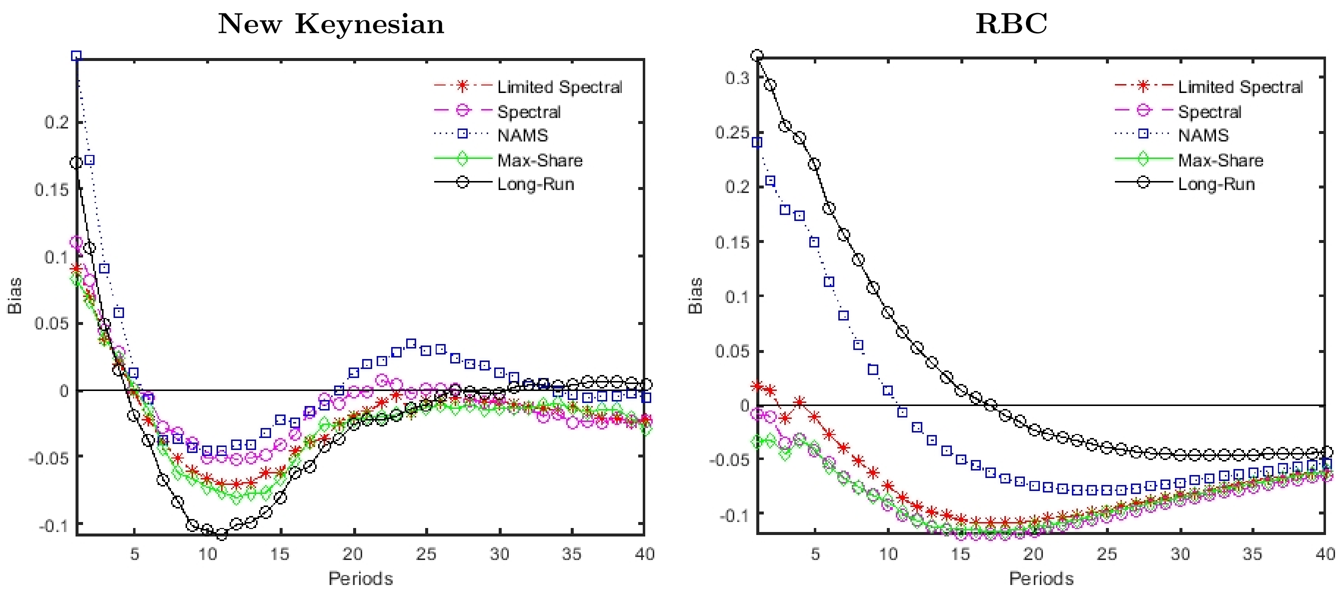
$$logZ_{t+1} = \mu_z + logZ_t + logz_{t+1}$$

$$\tau_{lt+1} = (1-\rho_l)\hat{\tau}_l + \rho_l \tau_{lt} + \epsilon_{lt+1}$$

The bias of the technology shock impact on hours

Many practitioners will be interested in the bias of each identification for the IRF of technology on hours worked. Figure 16 shows that for hours, a similar ranking of specifications occurs relative to the bias of the IRF of technology on productivity.

Figure 16: IRF bias of technology shock on hours worked: NK and RBC models



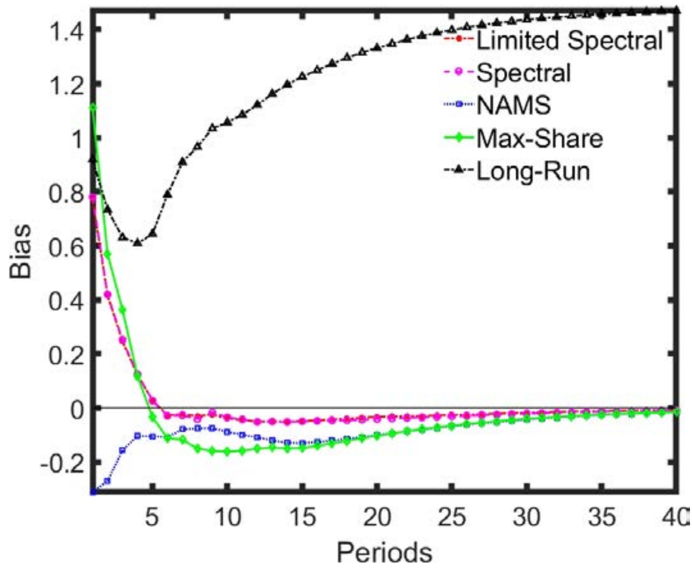
8.3 Alternative Simulations

8.3.1 More persistent confounding shocks

In section 4.2.2, the ability of each SVAR identification to correctly estimate a technology shock in the presence of confounding low-frequency shocks is evaluated. In this section, we evaluate the impact of making the confounding shock closer in persistence to the technology shock.

In the original scenario, the technology shock is assumed to have persistence 0.9, while the coefficient on the confounding low-frequency shock is 0.3 ($\rho_{b,1} = 0.3$). Here, we show that when raising the persistence of the confounding shock to 0.6, closer to the persistence of the technology shock (z). Figure 17 shows that the NAMS approach continues to have the lowest IRF bias even with the increased persistence of confounding shock, while the Spectral identification biases are the second-lowest in the initial stages of the IRF, consistent with the original scenario. In both cases however, the IRF bias has increased relative to the main-text scenario.

Figure 17: Bias of technology shock IRF for labor productivity: increased persistence of confounding low-frequency shock

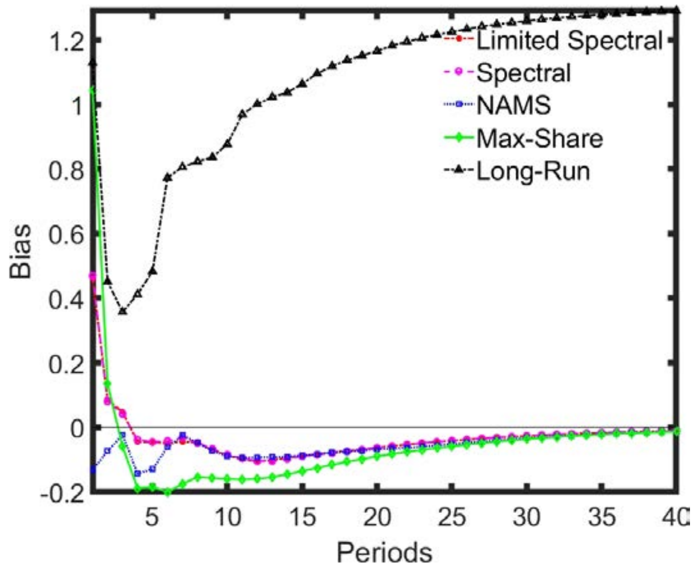


Note: Absolute bias of technology shock IRF for labor productivity compared to 'true' impulse, based on 100 simulations of data (250 periods in each simulation)

8.3.2 Changing the target horizon for Max-Share and NAMS

Simulations in this paper have used the standard target horizon for Max-Share of 10 years over which to maximize the forecast error variance (used by Francis et al. (2014) and Barsky and Sims (2011)). For consistency, we have adopted this time horizon for the NAMS approach. In this section, we evaluate the effects of increasing the target horizon for both NAMS and Max-Share to 15 years. IRF biases remain similar in magnitude to the baseline case of using a 10-year target range.

Figure 18: Bias of technology shock IRF for labor productivity: increased target horizon for Max Share and NAMS to 15 years

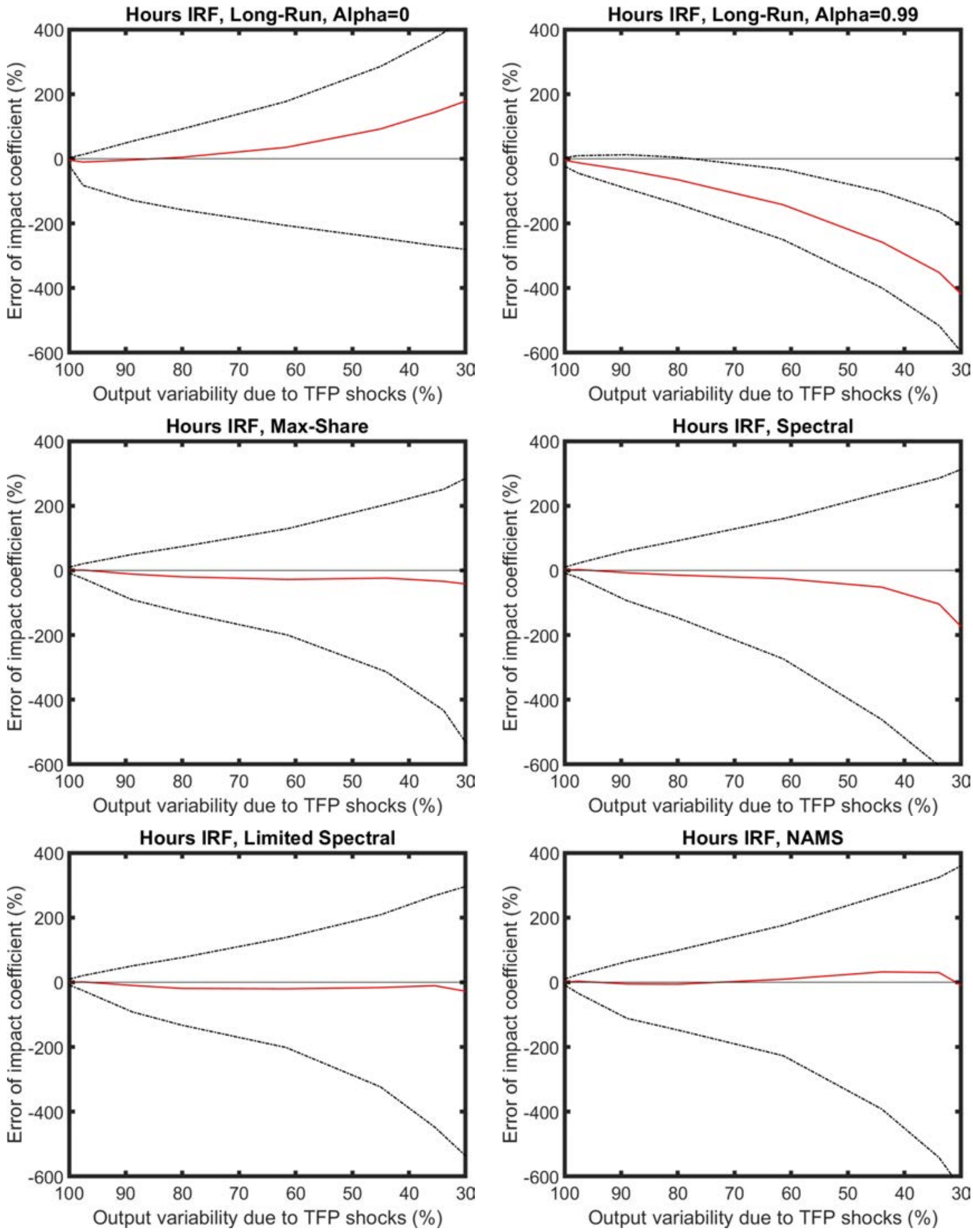


Note: Absolute bias of technology shock IRF for labor productivity compared to 'true' impulse, based on 100 simulations of data (250 periods in each simulation)

8.4 CKM (2008) simulations

In this appendix section, we first show that our new proposed specifications most accurately assess the technology shock's impact on hours as well as productivity. Second, we show that reducing the persistence of the non-technology shock from 0.95 (CKM's original parameterization) reduces the discrepancy in performance between the Max-Share and our proposed specifications - when these highly persistent shocks (with material effects on labor productivity at the 10-year horizon) are reduced, model performance is comparable.

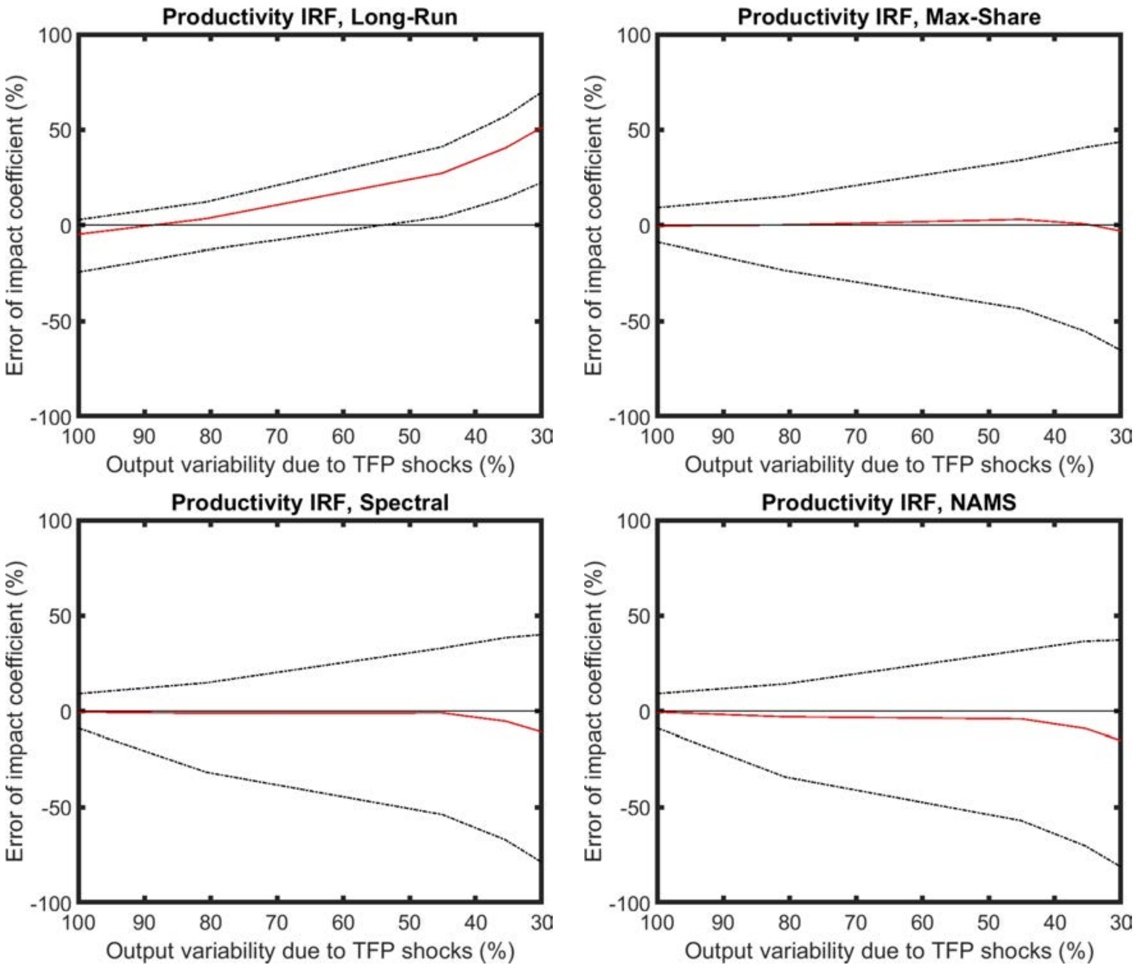
Figure 19: CKM: Impact bias on hours from technology as the proportion of output driven by non-technology shock is varied



Note: The proportion of variance driven by the non-technology shock is calculated by simulating the model with one shock at a time, and then comparing the variance of the HP-filtered series for output from each simulation, as in CKM.

In the case of the effect of a technology shock on hours, the Max-Share, Spectral, Limited Spectral and NAMs approaches show minimal bias-although the spectral approach bias does increase as technology shocks make up less than 50 percent of the variance of output (Figure 19). In the case of the long-run restriction, bias increases for lower influences of non-technology shocks. As noted in the main text, one of the reasons for the higher bias of the NAMS and spectral approaches in detecting the impact of technology on productivity in CKM is due to the high persistence of non-technology shocks in their model. As NAMS and the spectral methods are designed to capture highly persistent shocks, they are biased by the presence of multiple shocks with this characteristic. By reducing the persistence parameter of the non-technology shock from 0.95 to 0.7, the bias in both of these identifications falls considerably (Figure 20). This same change does not improve the performance of the long-run restriction.

Figure 20: CKM with lower persistence non-technology shock: Impact bias on productivity as proportion of output driven by non-technology shock is varied



Note: The proportion of variance driven by the non-technology shock is calculated by simulating the model with one shock at a time, and then comparing the variance of the HP-filtered series for output from each simulation, as in CKM. In this simulation, the persistence parameter of the non-technology shock is reduced from 0.95 to 0.7, demonstrating a lower bias for the NAMS and spectral approaches than in the high-persistence case

8.5 US Data Appendix

The application to the US data uses the following series from the St. Louis Fed's FRED database from 1953 Q2: 2018 Q3.

Table 7: VAR series and FRED database codes

Variable	FRED mnemonic and transformation
Labor productivity	log(OPHNFB)*100
Hours worked per capita	log(PRS85006023*CE16OV/CNP16OV)*100
Investment share (includes durable goods)	log((100*(PCDG+GPDI)/GDP))*100
Consumption share	log(100*(PCND+PCESV)/GDP)*100
Inflation	$(\Delta \log(\text{DPCERD3Q086SBEA}_t) * 100$
10-year treasury yield	GS10

References

Angeletos, George-Marios, F. C. and Dellas, H. (2020). Business-Cycle Anatomy. *American Economic Review*, October 2020(10):3030–70.

Barsky, R. B. and Sims, E. R. (2011). News shocks and business cycles. *Journal of Monetary Economics*, 58:273–289.

Blanchard, O. and Quah, D. (1989). The Dynamic Effects of Aggregate Demand and Supply Disturbances. *The American Economic Review*, 79(4):655–673.

Canova, F., Lopez-Salido, D., and Michelacci, C. (2010). The effects of technology shocks on hours and output: a robustness analysis. *Journal of Applied Econometrics*, 25(5):755–773.

Chari, V., Kehoe, P. J., and McGrattan, E. R. (2008). Are structural VARs with long-run restrictions useful in developing business cycle theory? *Journal of Monetary Economics*, 55(8):1337–1352.

Chari, V. V., Kehoe, P. J., and Mcgrattan, E. R. (2009). New Keynesian Models: Not Yet Useful for Policy Analysis. *American Economic Journal: Macroeconomics*, 1(1):242–266.

Christiano, L., Eichenbaum, M., and Vigfusson, R. (2003). What Happens After a Technology Shock? NBER working paper, National Bureau of Economic Research, Cambridge, MA.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113(1):1–45.

Christiano, L. J., Eichenbaum, M., and Vigfusson, R. (2007). Assessing Structural VARs. *NBER Macroeconomics Annual*, 21:1–106.

Christiano, L. J., Eichenbaum, M., and Vigfusson, R. J. (2006). Alternative procedures for estimating vector autoregressions identified with long-run restrictions. *Journal of the European Economic Association*, 4(2-3):475–483.

DiCecio, R. and Owyang, M. (2010). Identifying Technology Shocks in the Frequency Domain. Federal Reserve Bank of St. Louis Working Paper Series, Federal Reserve Bank of St. Louis, St. Louis.

Erceg, C. J., Guerrieri, L., and Gust, C. (2005). Can long-run restrictions identify technology shocks? *Journal of the European Economic Association*, 3(6):1237–1278.

Fisher, J. D. M. (2006). The Dynamic Effects of Neutral and Investment-Specific Technology Shocks. *Source: Journal of Political Economy*, 114(3):413–451.

Francis, N., Owyang, M. T., Roush, J. E., and Dicecio, R. (2014). A Flexible Finite-Horizon Alternative to Long-run restrictions with an application to technology shocks. *Review of Economic Statistics*, 96(4):638–647.

Francis, N. and Ramey, V. A. (2005). Is the technology-driven real business cycle hypothesis dead? Shocks and aggregate fluctuations revisited. *Journal of Monetary Economics*, 52:1379–1399.

Galí, J. (1999). Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations? *The American Economic Review*, 89(1):249–271.

Galí, J. and Rabanal, P. (2005). Technology Shocks and Aggregate Fluctuations: How Well Does the Real Business Cycle Model Fit Postwar U.S. Data? In *NBER Macroeconomics Annual 2004*, pages 225–318. National Bureau of Economic Research, Cambridge, MA.

Giannone, D., Lenza, M., and Reichlin, L. (2019). Money, Credit, Monetary Policy, and the Business Cycle in the Euro Area: What Has Changed Since the Crisis? *International Journal of Central Banking*, 15(5):137–173.

Kilian, L. (1998). Small-Sample Confidence Intervals for Impulse Response Functions. *The Review of Economics and Statistics*, 80(2):218–230.

Kurmann, A. and Sims, E. (2017). Revisions in Utilization-Adjusted TFP and Robust Identification of News Shocks. NBER Working Paper, National Bureau of Economic Research, Cambridge, MA.

Levchenko, A. and Pandalai-Nayar, N. (2018). TFP, News, and 'Sentiments': the International Transmission of Business Cycles. *Journal of the European Economic Association*, pages 1–40.

Lindé, J. (2008). The Effects of Permanent Technology Shocks on Labor Productivity and Hours in the RBC model. *Journal of Economic Dynamics and Control*, 33(3):597–613.

Medel, C. A. (2014). The typical spectral shape of an economic variable: a visual guide. *Applied Economics Letters*, 21(14):1017–1024.

Mertens, K. and Ravn, M. O. (2013). The dynamic effects of personal and corporate income tax changes in the United States. *American Economic Review*, 103(4):1212 – 47.

Sala, L. (2015). DSGE Models in the Frequency Domains. *Journal of Applied Econometrics*, 30(2):219–240.

Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3):586–606.

Uhlig, H. (2003). What moves real GNP? *mimeo HU Berlin*.

Uhlig, H. (2004). Do Technology Shocks Lead to a Fall in Total Hours Worked? *Journal of the European Economic Association*, 2(2-3):361–371.

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